Conjunction Assessment Risk Analysis

Conjunction Assessment Late-Notice High-Interest Event Investigation:

Space Weather Aspects

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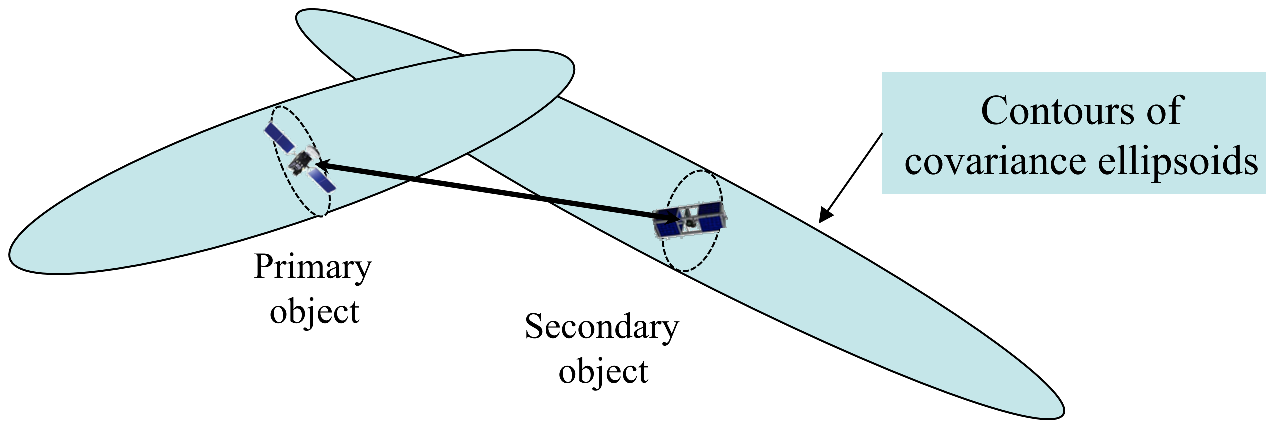
Background: Conjunction Assessment

- **Conjunction Assessment Risk Analysis (CARA)**
  - Evaluates collision risk between two satellites expected to come in close proximity of each other (by calculating probability of collision [Pc])
  - Mitigates collision risk, if necessary



- **Conjunctions usually identified several days before close approach**
  - Risk usually follows more-or-less canonical development paradigm
- **However, sometimes risk increases or decreases quite suddenly**
  - More insight needed into the circumstances behind such cases

Introduction

- **Tasked to analyze short notice events which are generally a result of unexpected, large state changes**
- **Looked at all reported conjunctions for *ca.* 700 km protected missions from May 2015 though Feb 2016**
- **Performed an analysis to determine whether there is any correlation between large state changes/late notice event identification and the following factors:**
  - Sparse tracking
  - High drag objects
  - Space weather
- **Examined specific late notice events identified by missions to try to identify root cause**

Broad Investigation of Large State Changes

- **Late-notice events usually driven by large changes in primary (protected) object or secondary object state**
- **Main parameter to represent size of state change is component position difference divided by associated standard deviation (ε/σ) from covariance**
- **Investigation determined actual frequency of large state changes, in both individual and combined states**
  - Compared them to theoretically expected frequencies
- **Found that large changes ($\varepsilon/\sigma > 3$) in individual object states occur much more frequently than theory dictates**
  - Effect less pronounced in radial components and in events with Pc > 1e-5
- **Found combined state matched much closer to theoretical expectation, especially for radial and cross-track**
  - In-track is expected to be the most vulnerable to modeling errors, so not surprising that non-compliance largest in this component

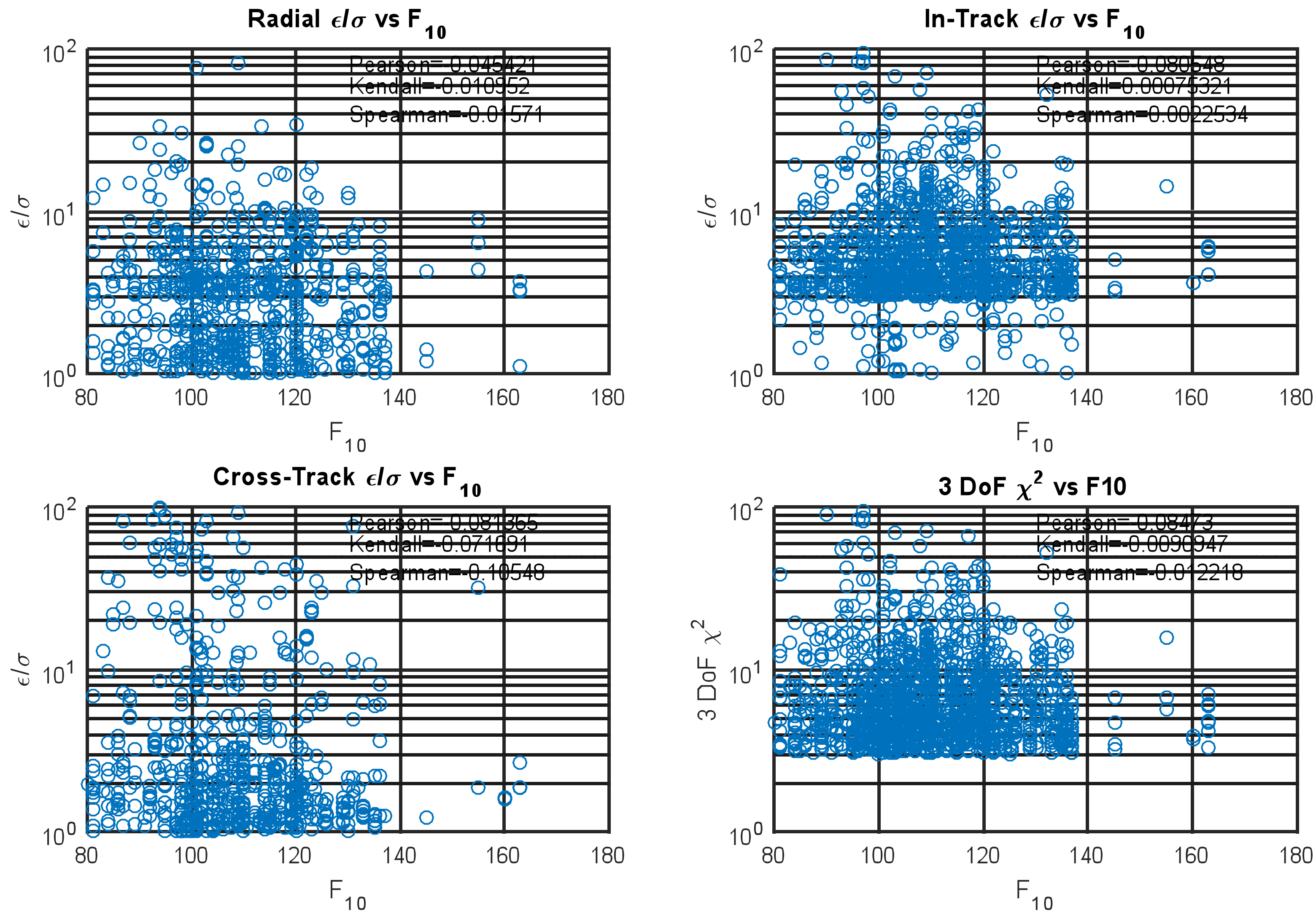
Summary of "Other" Correlation Results

- **Pc correlation with large state changes in primary not very strong**
- **Large state changes in the secondary do correlate to large changes in Pc, but not all that strongly**
  - Value of Kendall's Tau ranged from 0.37 to 0.6
- **Sparse tracking for secondary does not correlate with large state errors**
- **Higher EDR values for secondary do not correlate with larger state errors**

Correlations with Solar Activity

- **Elevated levels of solar activity can produce an unstable atmosphere whose density is difficult to model**
  - More strongly true with geomagnetic storms (Dst, a_p)
  - Can also be observed with EUV (F10, M10, S10, Y10, &c.)
- **Different possibilities for essence of the problem**
  - Higher solar activity *simpliciter*
  - Mismatch between predicted and realized solar activity
- **Will investigate the former with correlation studies**
  - Median F10 and a_p over prediction interval
  - Peak a_p over prediction interval
- **Will investigate the latter with case studies**

Combined ε/σ vs Median F_{10} : Any Component abs(ε/σ) > 3



Combined ε/σ vs Solar Indices: Tabular Summary

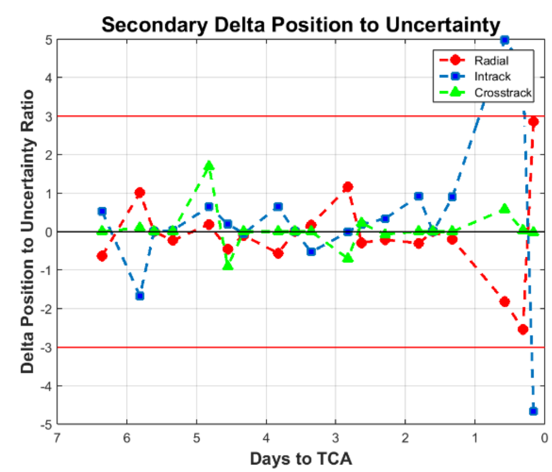
	Radial	In-Track	Cx-Track	Chi-Sq
Median F10: Kendall				
All Data	0.008	0.01	0.006	0.02
ε/σ > 3	0.01	0.001	0.07	0.01
ε/σ > 5	-0.03	-0.05	-0.09	-0.05
Median Ap: Kendall				
All Data	0.02	-0.0001	0.02	0.02
ε/σ > 3	-0.05	-0.01	0.06	-0.04
ε/σ > 5	-0.003	-0.003	0.03	0.01
Peak Ap: Kendall				
All Data	0.03	0.009	0.03	0.04
ε/σ > 3	-0.04	-0.01	0.03	-0.04
ε/σ > 5	-0.04	-0.01	-0.02	-0.04

- **Correlations are essentially nonexistent in all areas**

Simple elevated levels of solar activity do not correlate with large changes in relative miss

Late-Notice HIE Case Studies

- **Examined four late-notice events that fell within data investigation period of current study**
  - 1 MAY 2015 to 1 FEB 2016
- **Events examined**
  - Terra vs 38192, TCA 24 JUN 201
  - Aura vs 89477; TCA 29 AUG 2015
  - Terra vs 37131; TCA 19 DEC 2015
  - GPM vs 28685; TCA 5 SEP 2015
    - Determined not to be space weather related
- **Will look at**
  - ε/σ vs time (same as Δ position to uncertainty plots from daily/HIE report, like at right)
  - Pc vs time (same as from daily/HIE report)
  - Dst and a_p; prediction vs actual
    - Segmented by what is available in support of each update



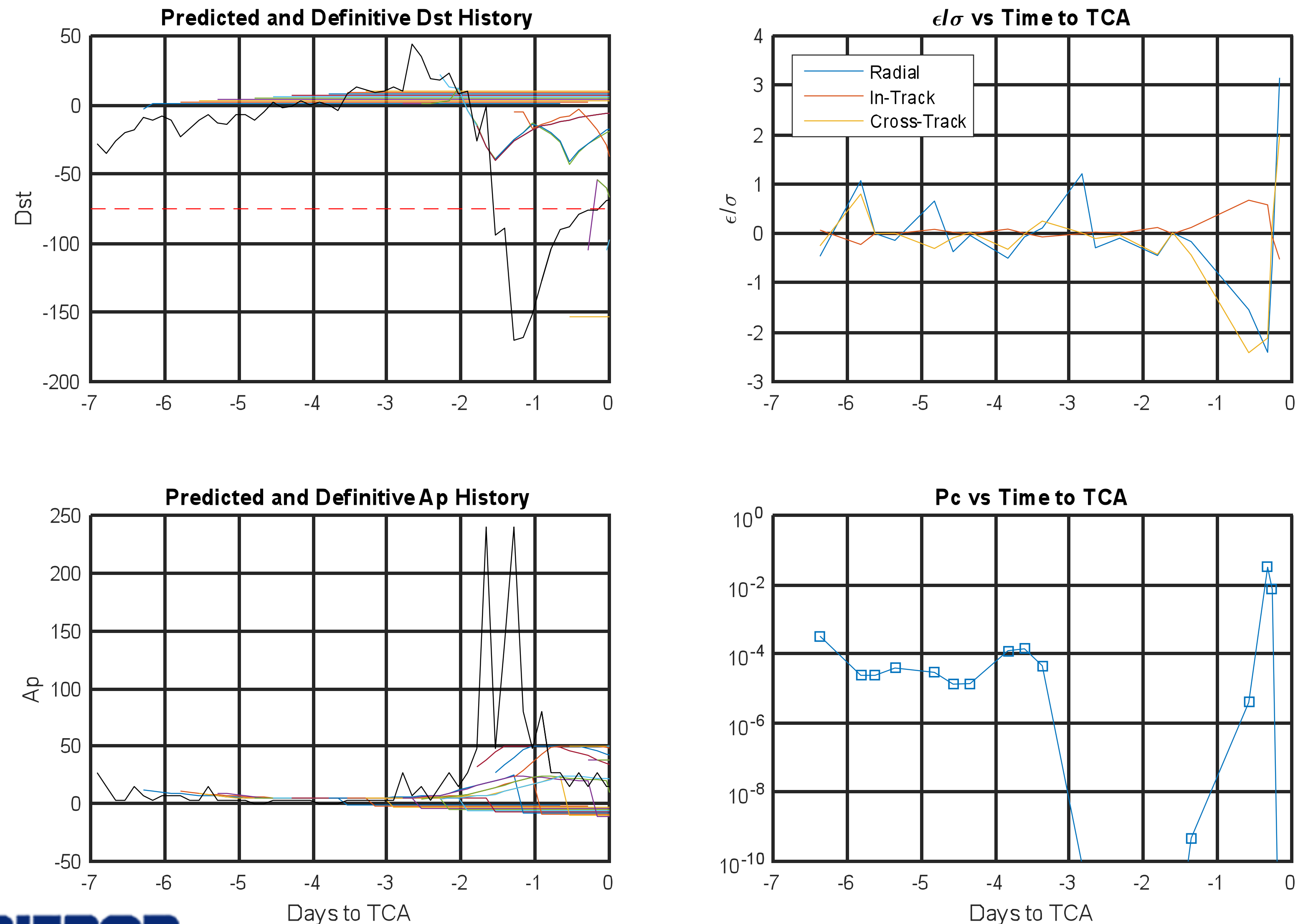
JSpOC Space Weather Information Files

- **JSpOC uses JBH09**
  - JB08 + HASDM
  - Anemomilos DST prediction
- **Updated at JSpOC 3x per day**
- **Model Input summary:**
  - **S10, S54** are daily and 54-day S10.7 index for >200 km heating of O by solar chromosphere 28.4-30.4 nm emissions in x10-22 Watts per meter squared per Hertz
  - **M10, M54** are daily and 54-day M10.7 index for 100-110 km heating of O2 by solar photosphere 160 nm SRC emissions in x10-22 Watts per meter squared per Hertz
  - **Y10, Y54** are daily and 54-day Y10.7 index for 85-90 km heating of N2, O2, H2O, NO by solar coronal 0.1-0.8 nm and Lya 121 nm emissions in x10-22 Watts per meter squared per Hertz
  - **F10, F54** are daily and 54-day solar 10.7 cm radio flux in x10-22 Watts per meter squared per Hertz
  - $\mathbf{a_p}$ is the 3-hour planetary geomagnetic 2 nT index (00-21 UT)
  - **Dst** is Disturbance Storm Time geomagnetic index in nT
  - **DTC** is delta exospheric temperature correction in units of K

Space Weather Evolution Charts

- **Upper left shows Dst; lower left shows a_p**
- **Black line is "issued" (definitive) data**
- **Colored lines are predicted data**
  - Each line begins when a given OD update executed
  - Each line shows predicted values of the geomagnetic index of choice
    - When Dst lines move to small positive value, prediction stops (zeroes in file)
    - When a_p lines move to small negative value, prediction stops (ones in file)
- **Dst threshold for solar storm compensation engagement also shown**
- **Upper right shows ε/σ for each component**
  - Miss distance vs combined covariance
- **Lower right shows Pc vs time**

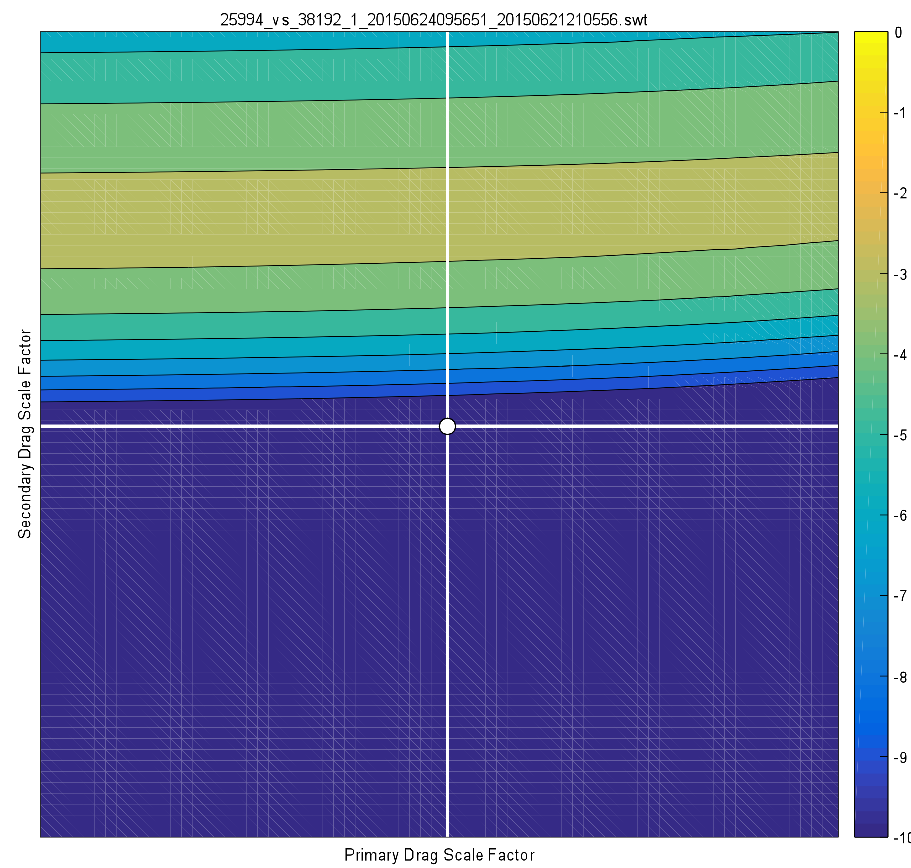
Case Study #1: Terra vs 38192, TCA 24 JUN 2015



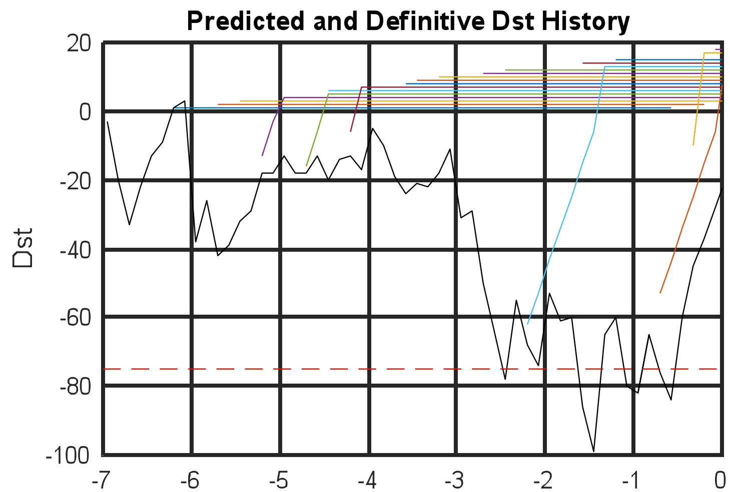
Space Weather Trade-Space Result: 61 Hours to TCA

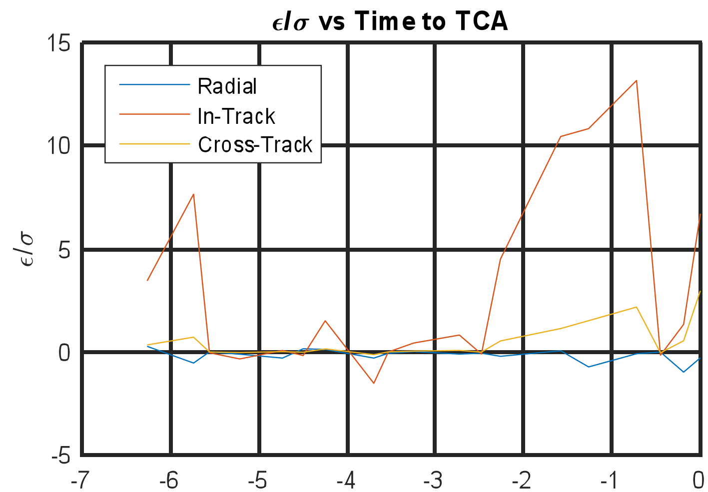
- **About half a day before spike in a_p/Dst begins**
  - Some predicted increased Dst activity, but not of severity actually realized
  - Predictions at very end of storm over-predict Dst
  - Final prediction and shrinking covariance produces Pc drop off
- **SWTS indicates conjunction vulnerable to large Pc changes due to density mis-modeling**

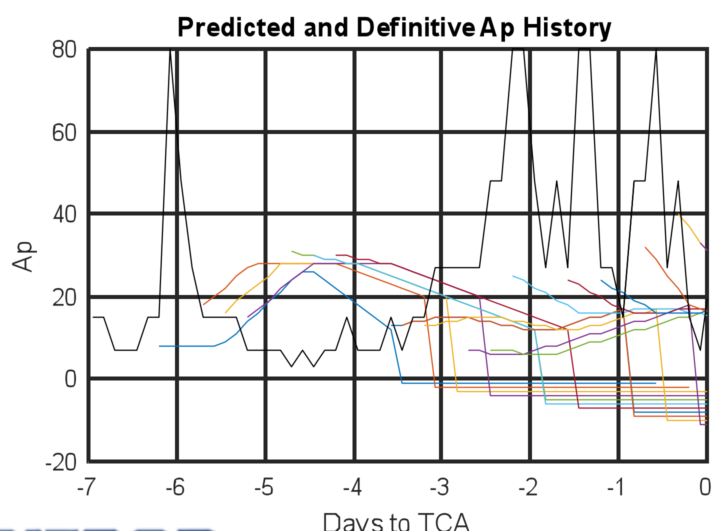
- **Bottom line: missed solar storm and subsequent prediction failures produced late changes**

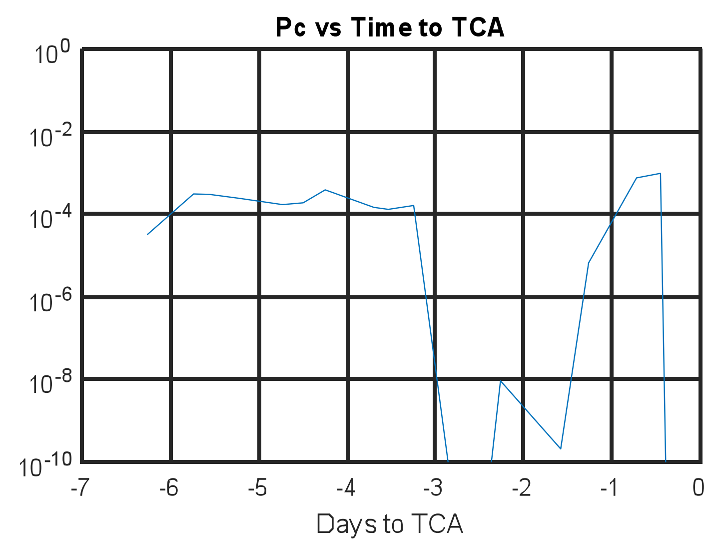


Case Study #2:
Aura vs 89477; TCA 29 AUG 2015



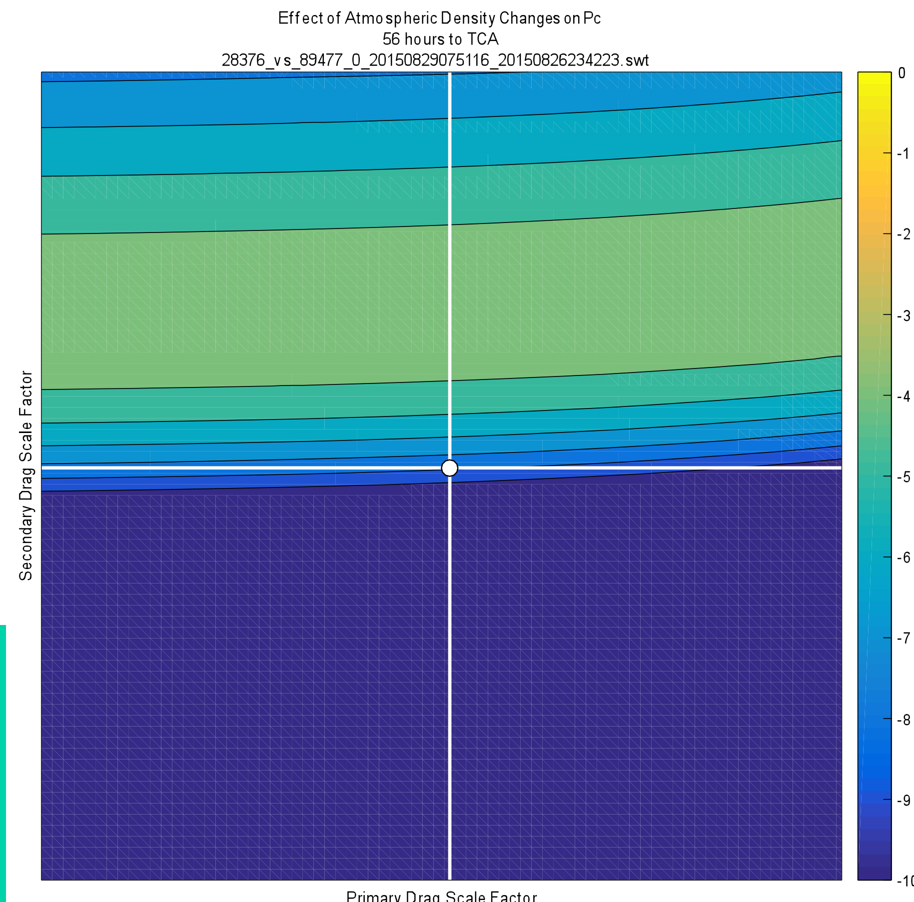






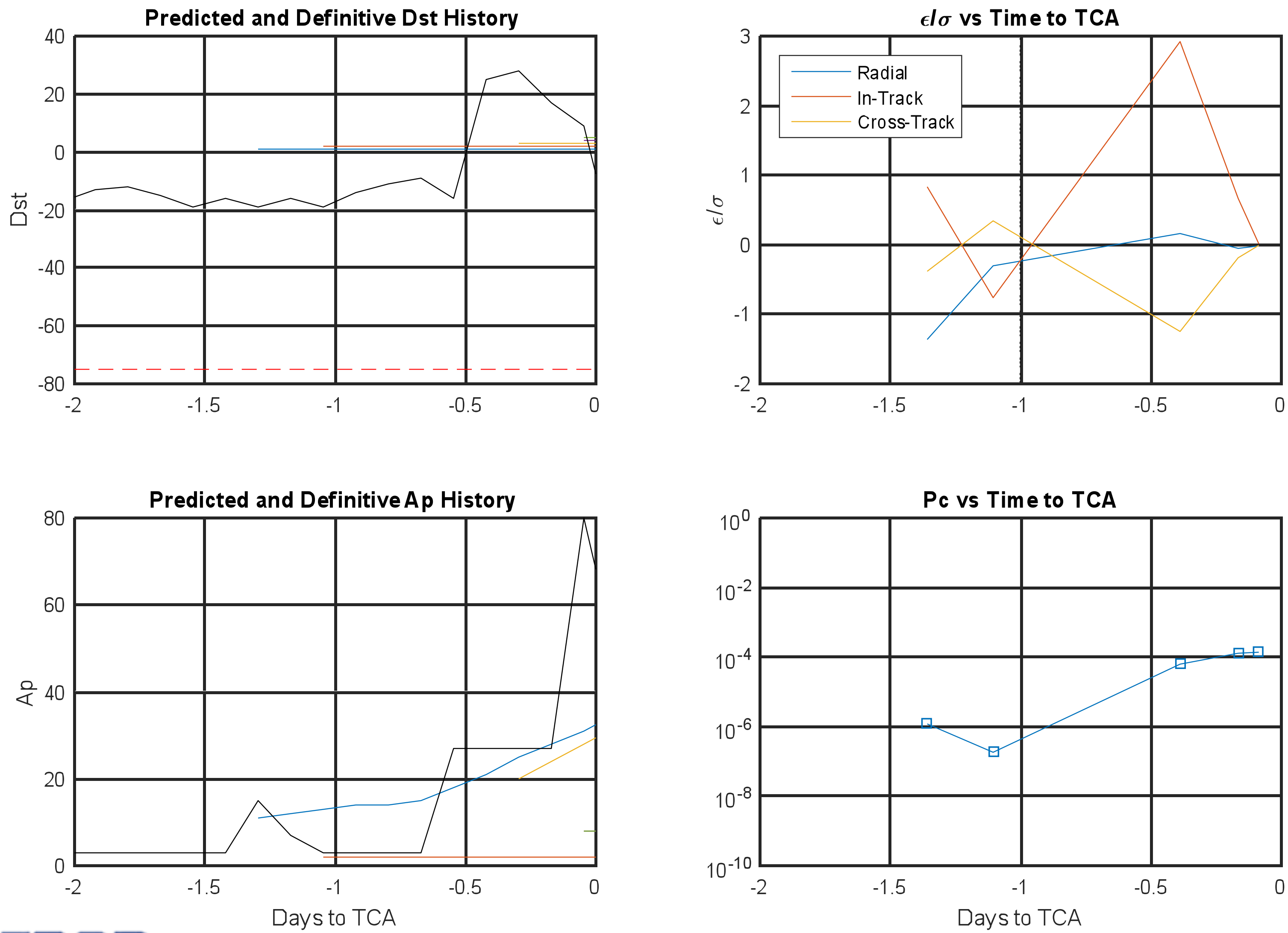
Space Weather Trade-Space Result: Aura vs 89477; 56 Hours to TCA

- **Run from update right as spike in a_p/Dst is beginning**
  - No predicted spike in relevant ASW space weather file
- **Indicates that conjunction vulnerable to large Pc changes due to atmospheric mis-modeling**



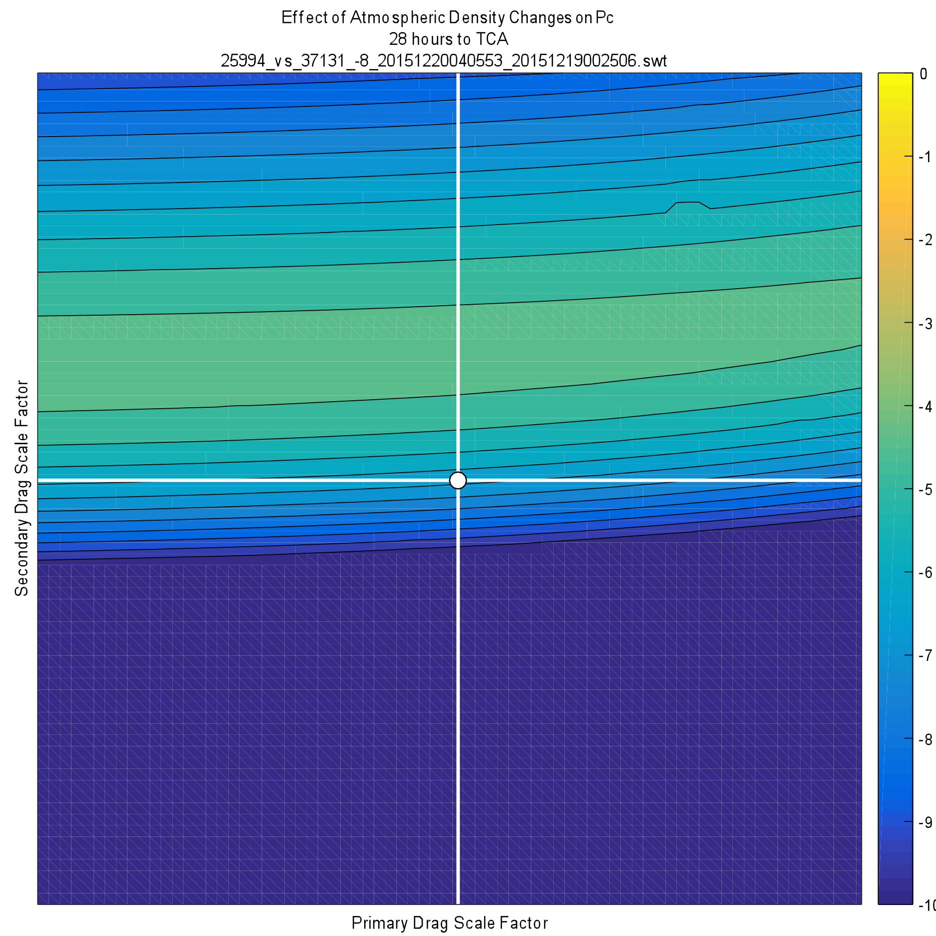
- **Bottom line: space weather predictions missed significant solar storm**
  - Most likely cause of late-breaking change in Pc

Case Study #3: Terra vs 37131; TCA 19 DEC 2015



Space Weather Trade-Space Result: Terra vs 37131; 28 Hours to TCA

- **Run from update before 2 OoM change in Pc observed**
  - Strange actual behavior in Dst
  - Modest unmodeled increase in Ap
- **SWTS indicates that conjunction vulnerable to Pc changes due to atmospheric mis-modeling**



- **Bottom line: odd space weather behavior, and deviation from predication, probably responsible for modest increase in Pc**

Late-Breaking HIEs: Overall Summary

- **Large state changes occur more often than theory would indicate**
- **Do not correlate at global level with any obvious causal condition**
  - Light tracking, hard-to-maintain orbits, or generally elevated solar activity
- **Case studies indicate two culprits**
  - Failure of JSpOC space weather predicted indices to predict solar storms
  - Edge cases for general screenings
- **Is there any good news?**
  - No, not really

Solar Storm Response – What are we doing?

- **CARA has begun receiving atmospheric model input data from JSpOC**
  - Gives CARA analysts insight into what is being modeled
  - CARA analysts can work with outside experts (SWRC) to evaluate reasonableness and likelihood of predicted space weather events
- **CARA analysts can use model input information and outside evaluation of predictions to provided more nuanced feedback as to when to expect increased uncertainty and variation due to space weather**
  - Additionally, as shown by this study, it is a great help for post-event analysis
- **Developing operational ConOps for how and when to apply space weather trade space with model insight**

BACKUP SLIDES

JSpOC Space Weather Information Files: Data Currency

- **Three types of data in file**
  - "Issued" – definitive values for the solar/geomagnetic index, subjected to full availability of feeder data and consistency tests
  - "Nowcast" – initial observations of values, hand-scaled and not subject to consistency tests
    - Measurements stay in "nowcast" status for typically 24 hours
  - "Predicted" – values are predicted
    - EUV predicted values from 54- and sometimes 108-day autoregression analyses of past data
    - Geomagnetic indices are predicted from observed solar activity earlier in the solar rotation (and thus expected to become georelevant at a given future time)
- **Data type timing**
  - Issued/Nowcast data used in propagating states from epoch to current time
    - Scaled/debiased with HASDM results
  - Predicted data used in propagating states from current time to TCA
  - Accuracy of predicted data can influence propagated result substantially

Normal Deviates and Chi-squared Variables

- **Let q and r be vectors of values that conform to a Gaussian distribution**
  - These collection of values are called *normal deviates*
- **A normal deviate set can be transformed to a *standard normal deviate* by subtracting the mean and dividing by the standard deviation**
  - This produces the so-called Z-variables

$$Z_q = \frac{q - \mu_q}{\sigma_q}, \quad Z_r = \frac{q - \mu_r}{\sigma_r}$$

- **The sum of the squares of a series of standard normal deviates produces a chi-squared distribution, with the number of degrees of freedom equal to the number of series combined**

$$Z_q^2 + Z_r^2 = \chi_{2\,dof}^2$$

Normal Deviates in State Estimation

- **In a state estimate, the errors in each component (u, v, and w here) are expected to follow a Gaussian distribution**
  - If all systematic errors have been solved for, only random error should remain
- **These errors can be standardized to the Z-formulation**
  - Mean presumed to be zero (OD should produce unbiased results), so no need for explicit subtraction of mean

$$Z_u = \frac{u}{\sigma_u}, \quad Z_v = \frac{v}{\sigma_v}, \quad Z_w = \frac{w}{\sigma_w}$$

- **Sum of squares of these standardized errors should follow a chi-squared distribution with three degrees of freedom**

$$Z_u^2 + Z_v^2 + Z_w^2 = \chi^2_{3dof}$$

State Estimation Example Calculation

- **Let us presume we have a precision ephemeris, state estimate, and covariance about the state estimate**
  - For the present, further presume covariance aligns perfectly with uvw frame (no off-diagonal terms)
- **Error vector ε is position difference between state estimate and precision ephemeris, and covariance consists only of variances along the diagonal**
  - Inverse of covariance matrix is straightforward

$$\varepsilon = \begin{bmatrix} \varepsilon_u \\ \varepsilon_v \\ \varepsilon_w \end{bmatrix}, \quad C = \begin{bmatrix} \sigma_u^2 & 0 & 0 \\ 0 & \sigma_v^2 & 0 \\ 0 & 0 & \sigma_w^2 \end{bmatrix} \qquad C^{-1} = \begin{bmatrix} 1/\sigma_u^2 & 0 & 0 \\ 0 & 1/\sigma_v^2 & 0 \\ 0 & 0 & 1/\sigma_w^2 \end{bmatrix}$$

- **Resultant simple formula for chi-squared variables**

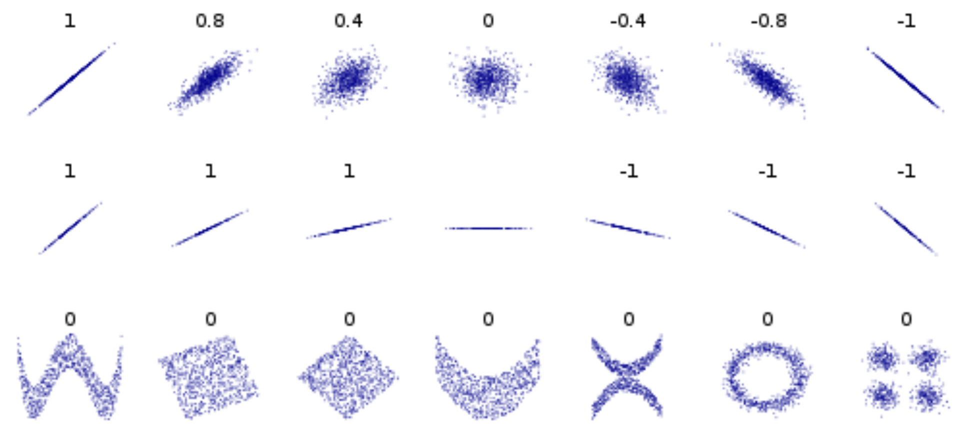
$$\varepsilon C^{-1} \varepsilon^T = \frac{\varepsilon_u^2}{\sigma_u^2} + \frac{\varepsilon_v^2}{\sigma_v^2} + \frac{\varepsilon_u^2}{\sigma_w^2} = \chi^2_{3\,dof}$$

- **Extension to case with off-diagonal terms straightforward**

Pearson Correlation Coefficient

- **Evaluates the degree of a linear relationship between two variables**
- **Usually evaluated by the formula (s is sample standard deviation), with range of interesting and often not helpful outcomes**

$$r = \frac{1}{n-1}\sum_{i=1}^{n}\left(\frac{x_i - \bar{x}}{s_x}\right)\left(\frac{y_i - \bar{y}}{s_y}\right)$$



- **Some interpretive guidance via relationship to r^2 value from linear regression: square of Pearson = regression r^2**
  – Pearson value of 0.5 would equate to r^2 of 0.25—not very impressive
- **Really would like something that reveals even non-linear correlation**

Kendall's Tau

- **Rank correlation test**
  - With two vectors of data X and Y, compares (Xi,Yi) to every other (Xj,Yj)
  - Pair is concordant if, when Xi>Xj, Yi>Yj; discordant if the opposite
  - Parameter is (# concordant pairs - # discordant pairs) / (total pairs)
    - So same range of values (-1 to 1) with same meaning
- **Much more robust test**
  - Will find both linear and nonlinear correlation
  - Computationally expensive [~$O(n^2)$], but computers are doing the work
- **Tied situations create problems**
  - In present analysis, arises when comparing continuous to discrete distribution
    - *e.g.,* ε/σ to tracking levels (because tracking levels are counting numbers, so can have multiple ε/σ values aligned with same tracking level)
  - Even more computationally expensive modifications to adjust for ties
  - Spot-checked these and saw no difference in computed result

Spearman's Rho

- **Test of monotonicity, computed by summing squares of differences in rank**
  - Mapped into same -1 to 1 range of values, with same interpretation
- **Computational formula**

$$\rho = 1 - \frac{6\sum_{i=1}^{n} d_i^2}{n\left(n^2 - 1\right)}$$

- **Computationally easier but more vulnerable to outlier data**
- **Usually larger than Kendall's tau**
- **Included here for consistency/contrast**

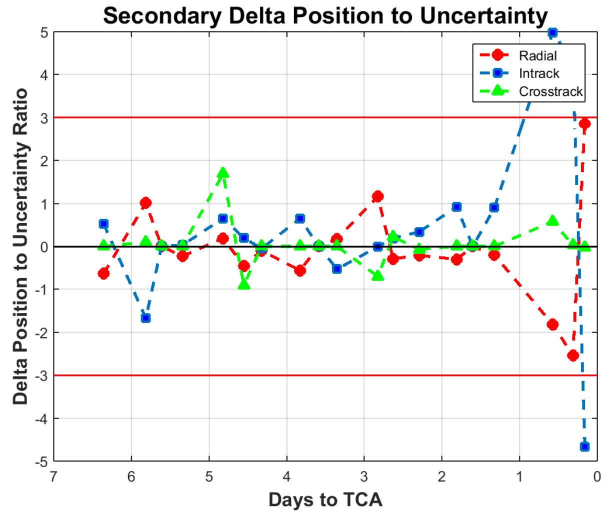
Main factor to consult is Kendall's Tau

Broad Investigation of Large State Changes

- **Determine actual frequency of large state changes, in both individual and combined states**
  - Compare to theoretically expected frequencies
- **Determine whether broadly correlated with potential/expected causes**
  - Low tracking
  - Harder-to-maintain orbits (larger energy dissipation rate)
  - General levels of solar activity (EUV and Joule atmospheric heating)

Large State Changes: Parameterization (1 of 3)

- **Main parameter to represent size of state change is component position difference divided by associated standard deviation (ε/σ)**
  - Presumption of OD is that errors are normally distributed and unbiased
  - ε is difference in component position between subsequent state estimates
  - σ is square root of associated variance from first state's covariance
  - Dividing ε by σ creates standardized normal variable (μ=0 because unbiased)
  - Set of these should thus conform to standard normal distribution
- **Same method currently used in CARA daily and HIE reports**



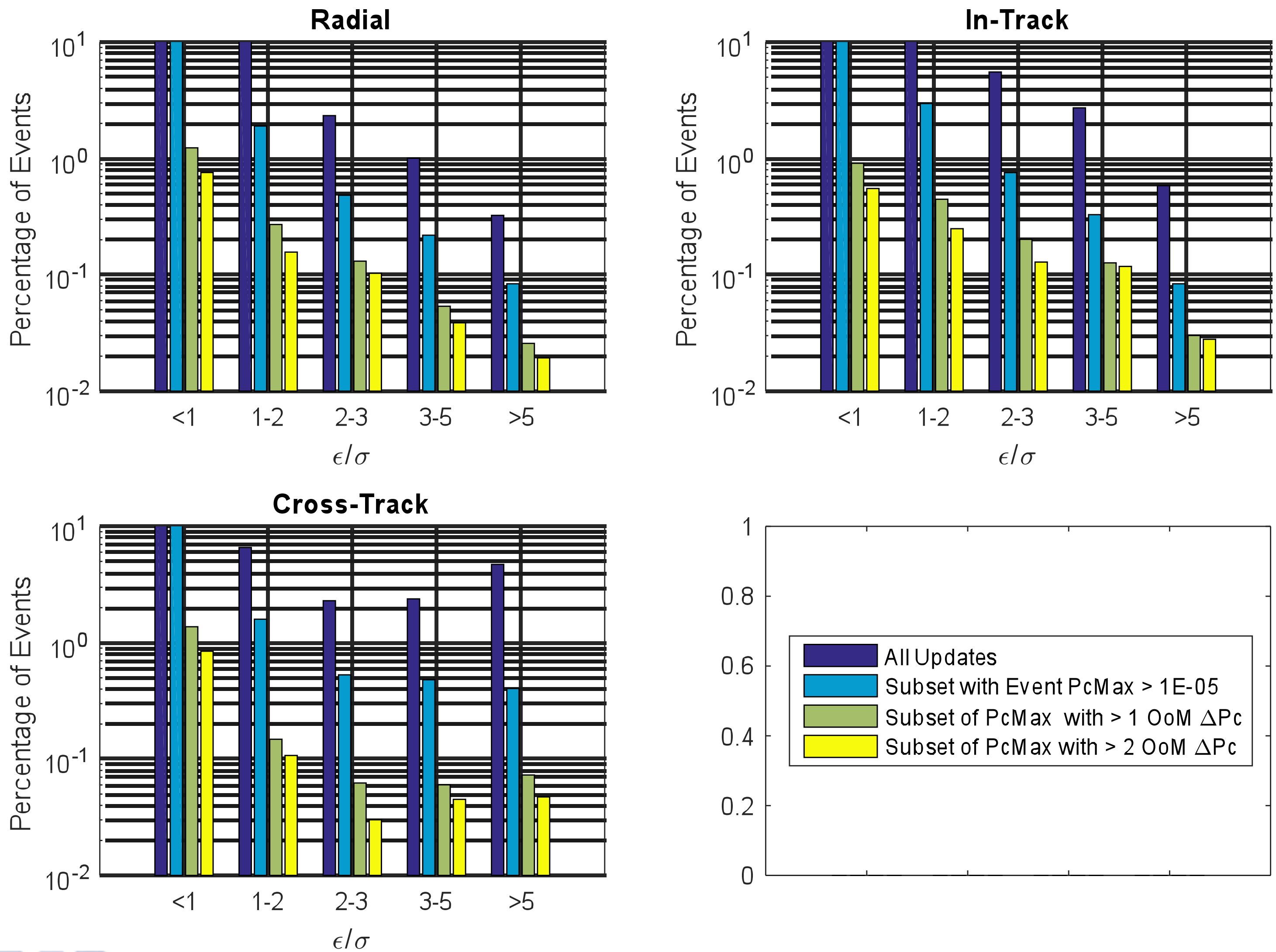
Large State Changes: Parameterization (2 of 3)

- **However . . . This is only true for the “diagonalized” situation, in which covariance axes and coordinate frame axes align**
  - Results meaningful only if ellipse closely aligns with coordinate axes
  - Once ellipse rotated, then component errors are correlated
    - Individual component error distributions no longer independent random variables
- **How often are covariance error ellipsoids naturally diagonalized?**
  - Not terrible assumption for individual satellites (primary, secondary)
  - More tenuous for combined situation (miss distance vs combined covariance)
- **Bottom line: ε/σ statistics at the component level must be used with care**
  - When plotted against only positive axis, presume ε/σ to be abs(ε/σ)

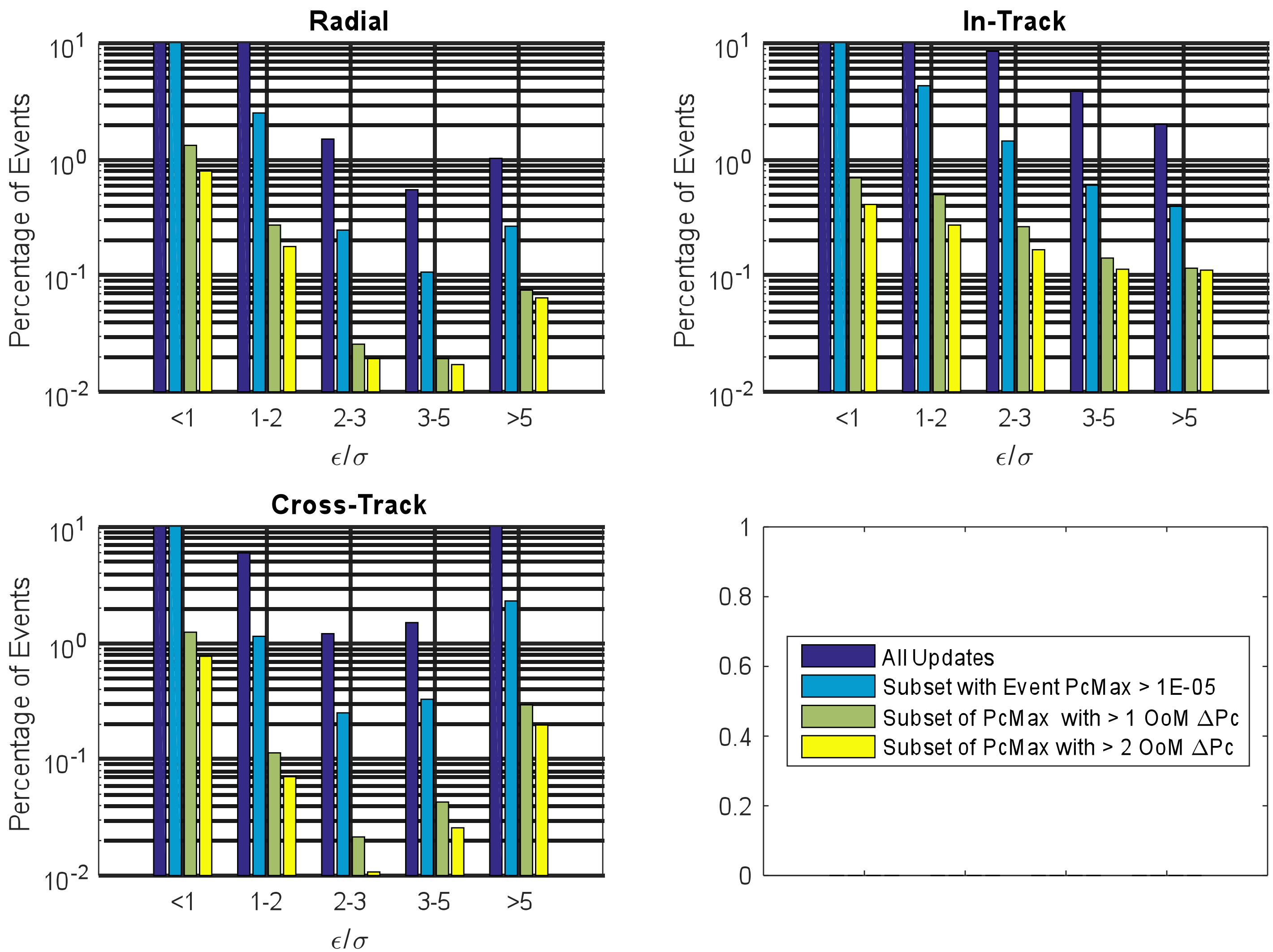
Large State Changes: Parameterization (3 of 3)

- **Comparison alternative: Mahalanobis distance**
  - If individual component errors normally distributed, then sum of squares of individual ratios (ε^2/σ^2) will constitute a 3-DoF χ^2 distribution
  - Formulary $\varepsilon C^{-1} \varepsilon^T$ properly considers all correlations and makes the calculation independent of coordinate system
  - Approach less frequently encountered, so less intuition built up around result
  - But will be supplied and examined along with Gaussian variables
  - Can also examine 2-DoF situation for only radial and in-track
    - More information on this later

Frequency of Large State Changes: Secondary Objects



Frequency of Large State Changes: Primary Objects



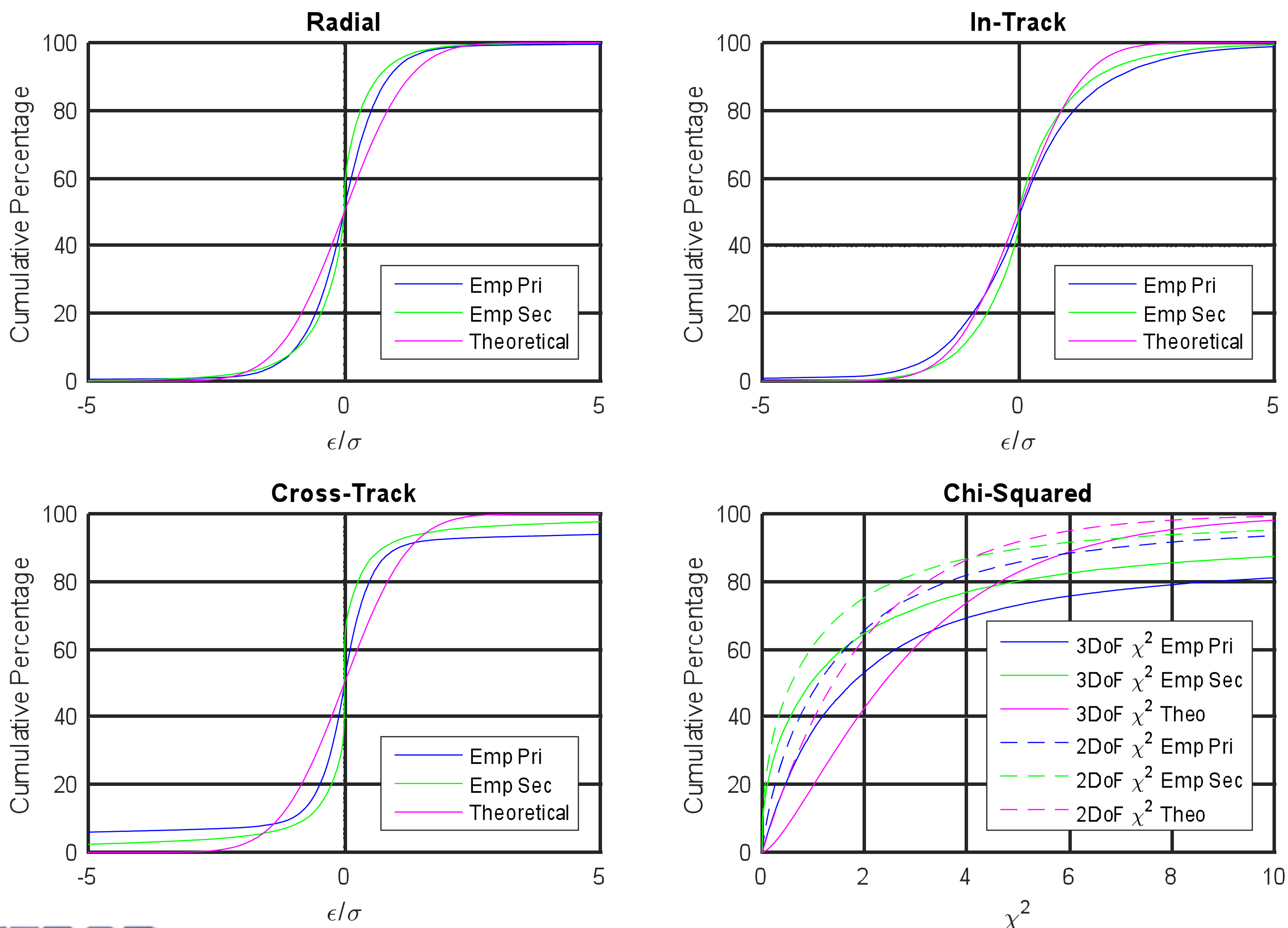
Summary of Frequencies: Primary and Secondary Objects

- **Data summary**
  - Table below reports situation for which abs(ε/σ) > 3
- **Commonly-known theoretical "percentages" for univariate Gaussian distribution consider two-tailed results**
  - 95.4% for 2-σ distribution considers results from 2.3% to 97.7%
  - 99.7% for 3-σ distribution considers results from 0.15% to 99.85%
- **Actual percentages for primaries surprisingly large**
  - Very similar for radial component; much larger differences with other two
    - Perhaps a little comfort in this, as radial generally most important component for CA

	Percent of Events in which abs(ε/σ) exceeds 3							
	Overall		Event Pc > 1E-05		> 1E-05 & ΔPc > 1 OoM		> 1E-05 & ΔPc > 2 OoM	
	Primary	Secondary	Primary	Secondary	Primary	Secondary	Primary	Secondary
Radial	1.57	1.33	0.37	0.30	0.09	0.08	0.08	0.06
In-Track	5.88	3.31	1.00	0.41	0.26	0.16	0.23	0.15
Cross-Track	13.53	7.10	2.64	0.88	0.34	0.13	0.22	0.09

Overall, prevalence is greater than theory would predict. However, presence in events of significance notably reduced

Comparison of ε/σ to Theory: Primary and Secondary Objects



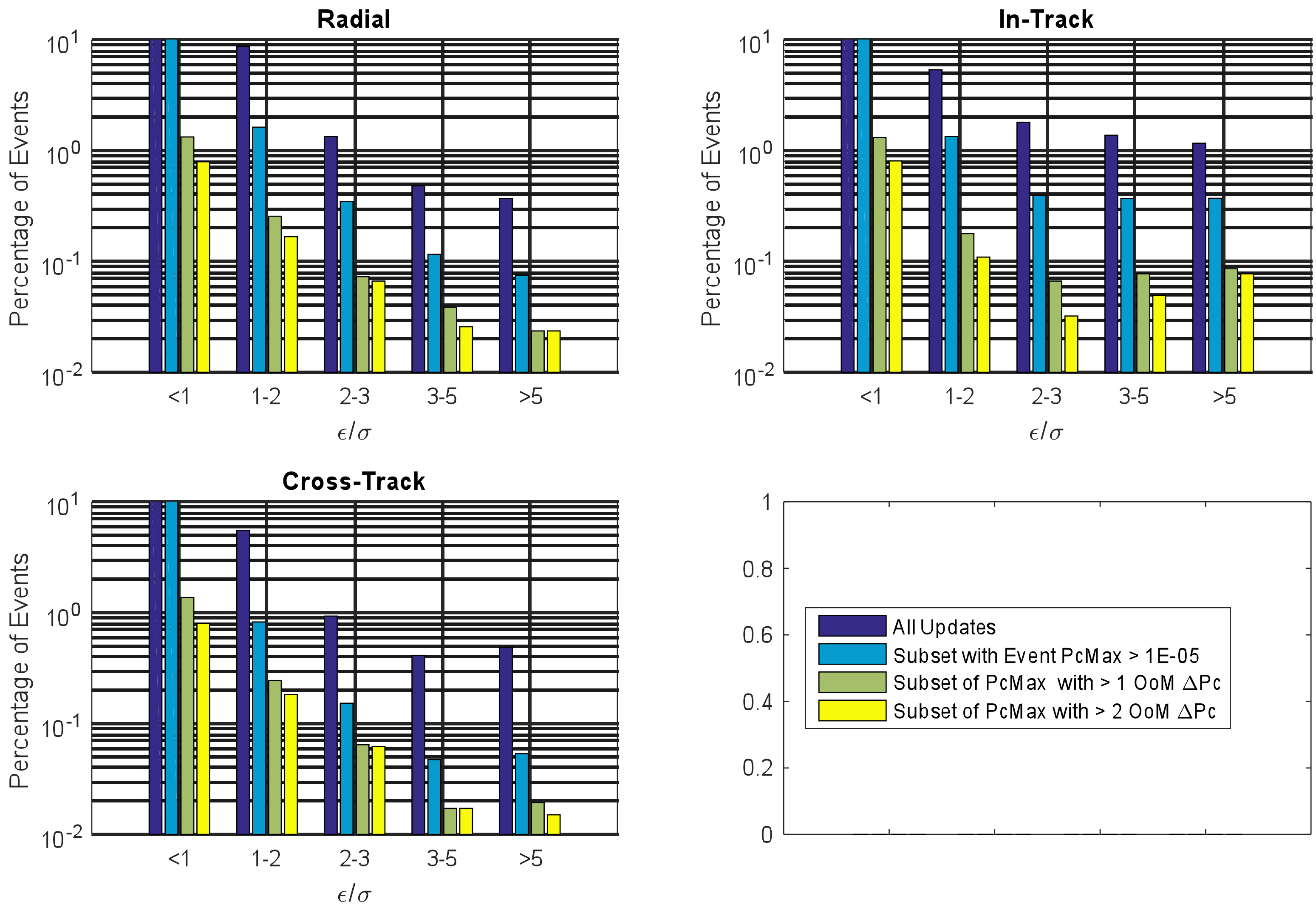
Comparison of ε/σ to Theory: Interpretation

- **Radial behaves reasonably well—better than theory until more extreme part of tails reached**
  - Cannot see tail behavior very well in provided plots
- **In-track has non-theoretical distribution beyond about ε/σ > 1**
  - As remarked previously, worse for secondaries than for primaries
- **Cross-track highly leptokurtic—peaked with very long tails**
  - Does not match a Gaussian distribution at all
- **In using chi-squared distribution, 2-DoF framework gives more sanguine situation**
  - Eliminates effect of large cross-track differences
  - Nonetheless, non-theory outliers dominate performance in the tails
- **None of these results sets match the theory particularly well**
- **Immediate conclusion difficult**
  - OD residuals suspected to be leptokurtic
  - Present trend could be extension of this

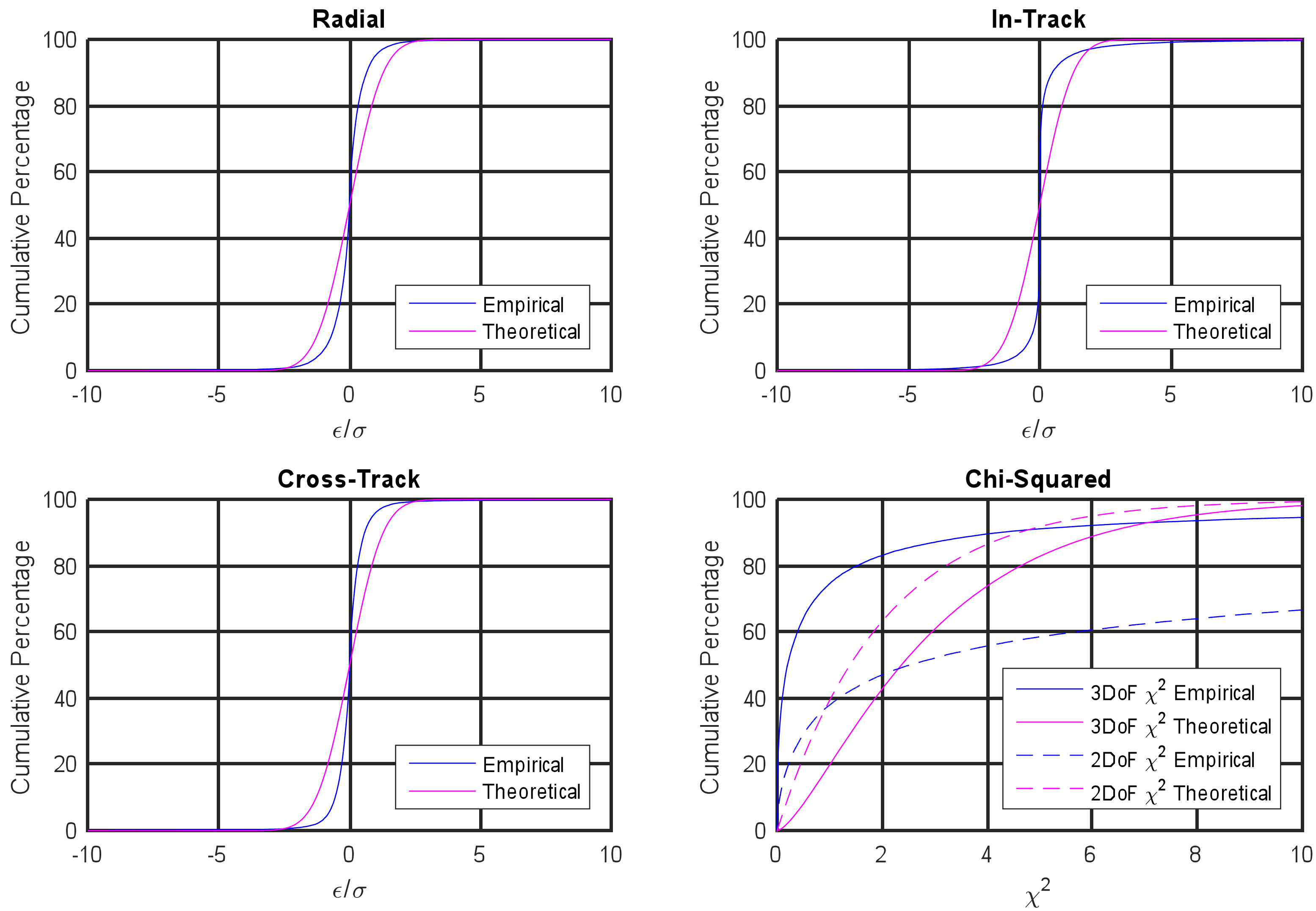
Combined Situation

STATE-CHANGE FREQUENCY AND COMPARISON TO THEORY

Frequency of Large State Changes: Miss vs Combined Sigma



Comparison of ε/σ to Theory: Miss Component vs Combined Sigma

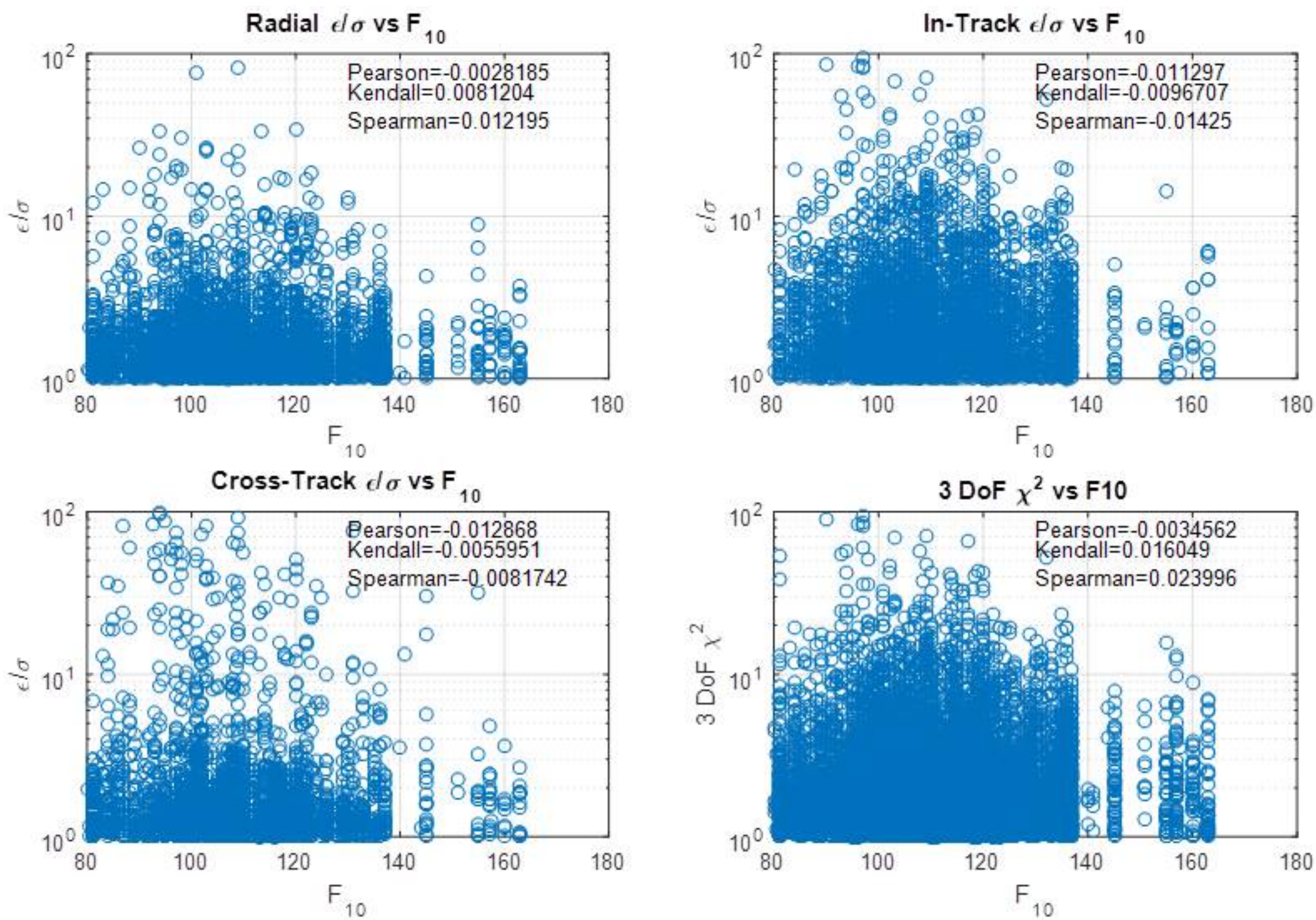


Frequency of Large State Changes: Tabular Summary

	Percent of Events in which abs(ε/σ) exceeds 3											
	Overall			Event Pc > 1E-05			> 1E-05 & ΔPc > 1 OoM			> 1E-05 & ΔPc > 2 OoM		
	Primary	Secondary	Combined	Primary	Secondary	Combined	Primary	Secondary	Combined	Primary	Secondary	Combined
Radial	1.57	1.33	0.85	0.37	0.30	0.19	0.09	0.08	0.06	0.08	0.06	0.05
In-Track	5.88	3.31	2.54	1.00	0.41	0.74	0.26	0.16	0.16	0.23	0.15	0.13
Cross-Track	13.53	7.10	0.90	2.64	0.88	0.10	0.34	0.13	0.04	0.22	0.09	0.03

- **Values much closer to theoretical expectation, especially for radial and cross-track**
  - In-track is expected to be the most vulnerable to modeling errors, so not surprising that non-compliance largest in this component

Combined ϵ/σ vs Median F_{10}: All Data



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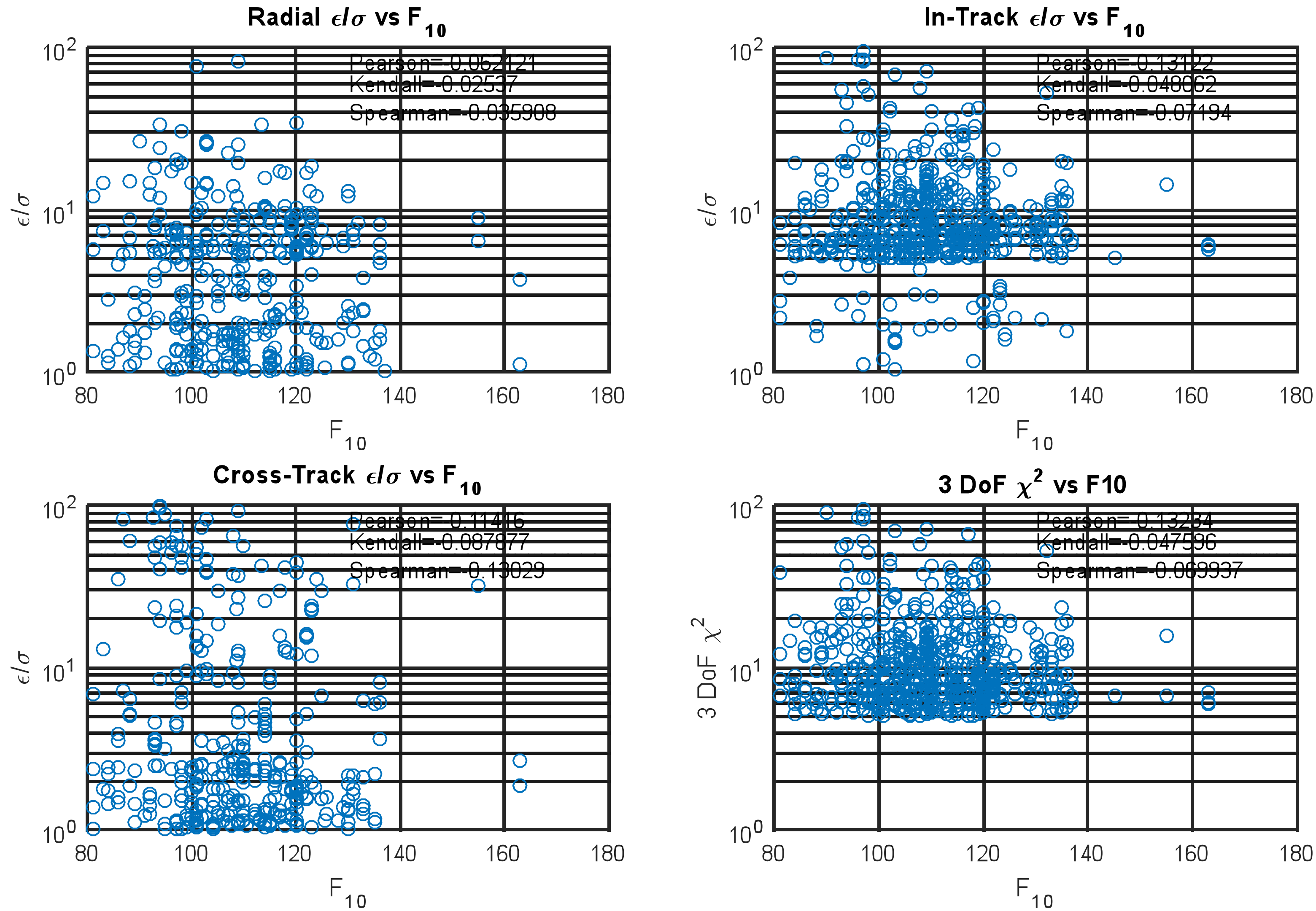
Combined ε/σ vs Median F_{10} : Any Component abs(ε/σ) > 5



Issues in Comparison to Theory

- **Commonly-known "percentages" for univariate Gaussian distribution consider two-tailed results**
  - 95.4% for 2-σ distribution considers results from 2.3% to 97.7%
  - 99.7% for 3-σ distribution considers results from 0.15% to 99.85%
- **Potential double-counting of large state changes**
  - Subsequent updates analyzed for large state change behavior
  - In a chain of updates, return to normalcy will appear as a second large change
  - Demarcation between one and two events not so easy to define (S = small state change; L = large state change
    - S S L L S S – one or two events?
    - S S S L S S L S S – one or two events?
    - S S S S S S L – one or two events (would it have been counted as two if one more update had been available?
  - For data-mining simplicity, all large changes counted, with the caveat that reported number might be twice as large as "actual" number

Solar Storm Predictions: What are we Doing? (1 of 2)

- **CARA member of NASA LWS space weather expert panel**
  - Dr. Matt Hejduk as CA expert panel representative
  - Dr. Yihua Zheng as GSFC space physics representative, also representing mission interests
- **Purpose of panel to recommend NASA research investments to improve prediction and modeling**
  - Will issue formal report of recommendations by December, as well as accompanying journal article
  - Will attempt to focus at least part of recommendation to address JSpOC situation
- **Hope to leverage report to push state of the art at JSpOC**
  - However, from their perspective, a large investment was just made in atmospheric density prediction modeling; need to focus on other items

Solar Storm Predictions: What are we Doing? (2 of 2)

- **Will investigate whether file update frequency can be accelerated**
  - Brief JSpOC on these results to show the problems that latencies create
    - See if there are mechanisms to improve efficiencies
  - Use SWTS function to determine whether such intervention is needed
    - Events that are not vulnerable to atmospheric density mismodeling would not require out-of-cycle updates
  - Would not have helped cases investigated here, as entire solar storms were missed
- **However, probably a fairly long time before there is much improvement with such scenarios**