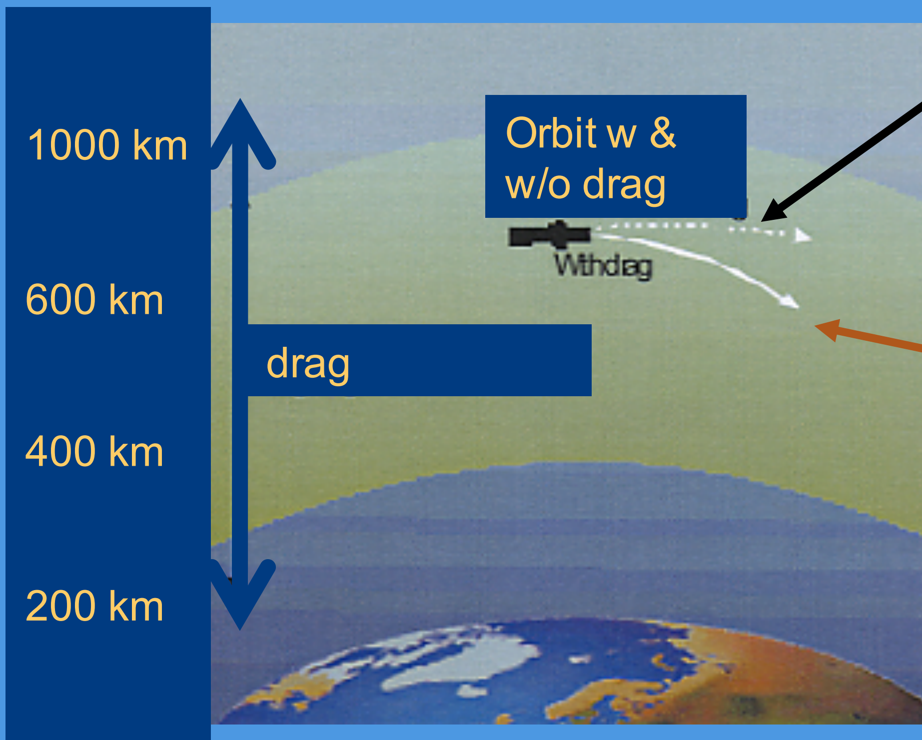
Orbit determination / extrapolation

Sean Bruinsma

Orbit computation: force model

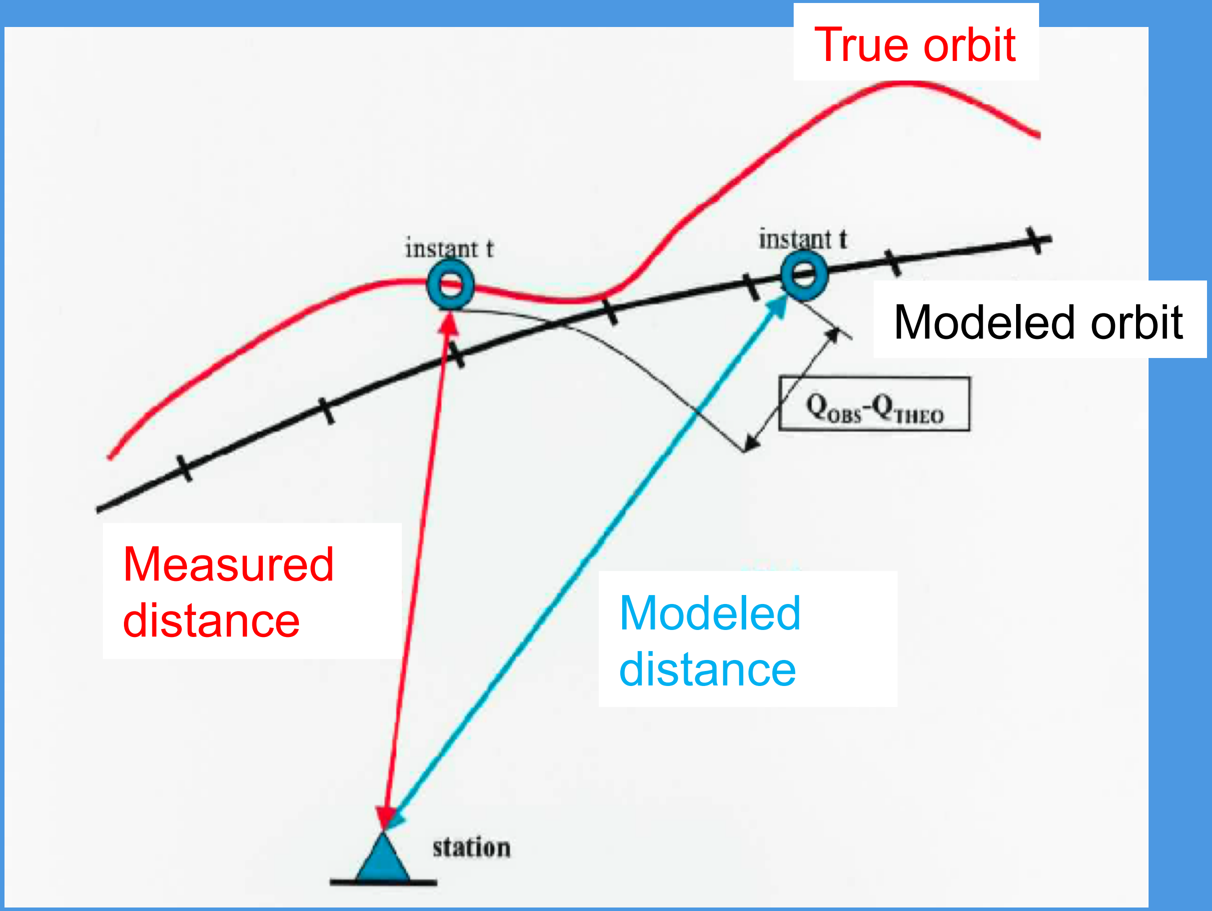


Gravitational forces:
- gravity (earth, moon, sun, planets);
- ocean & solid earth tides.

Surface forces:
- radiation pressure (sun, earth)
- ***atmospheric drag***

Effect: The semi-major axis (altitude) of the orbit decreases

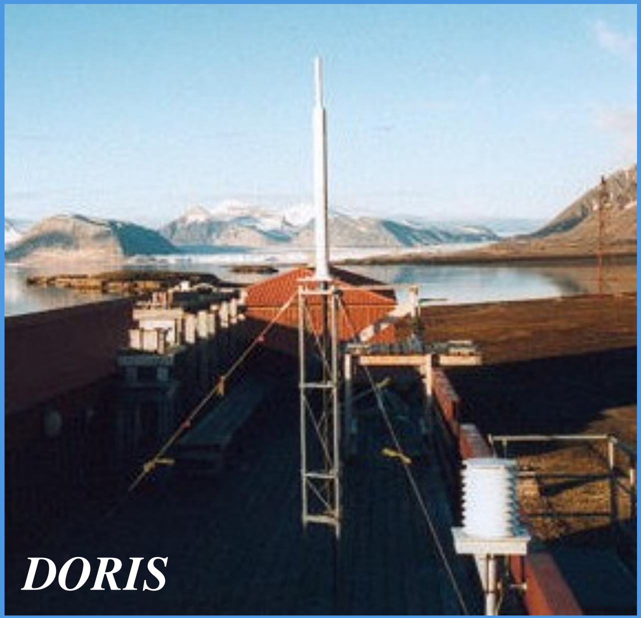
Perturbation analysis



The orbit residuals (Measured – Modeled) reflect the accuracy of the force model, and enable the estimation of corrections to specific models (e.g., drag scale factor)

Geodetic tracking techniques

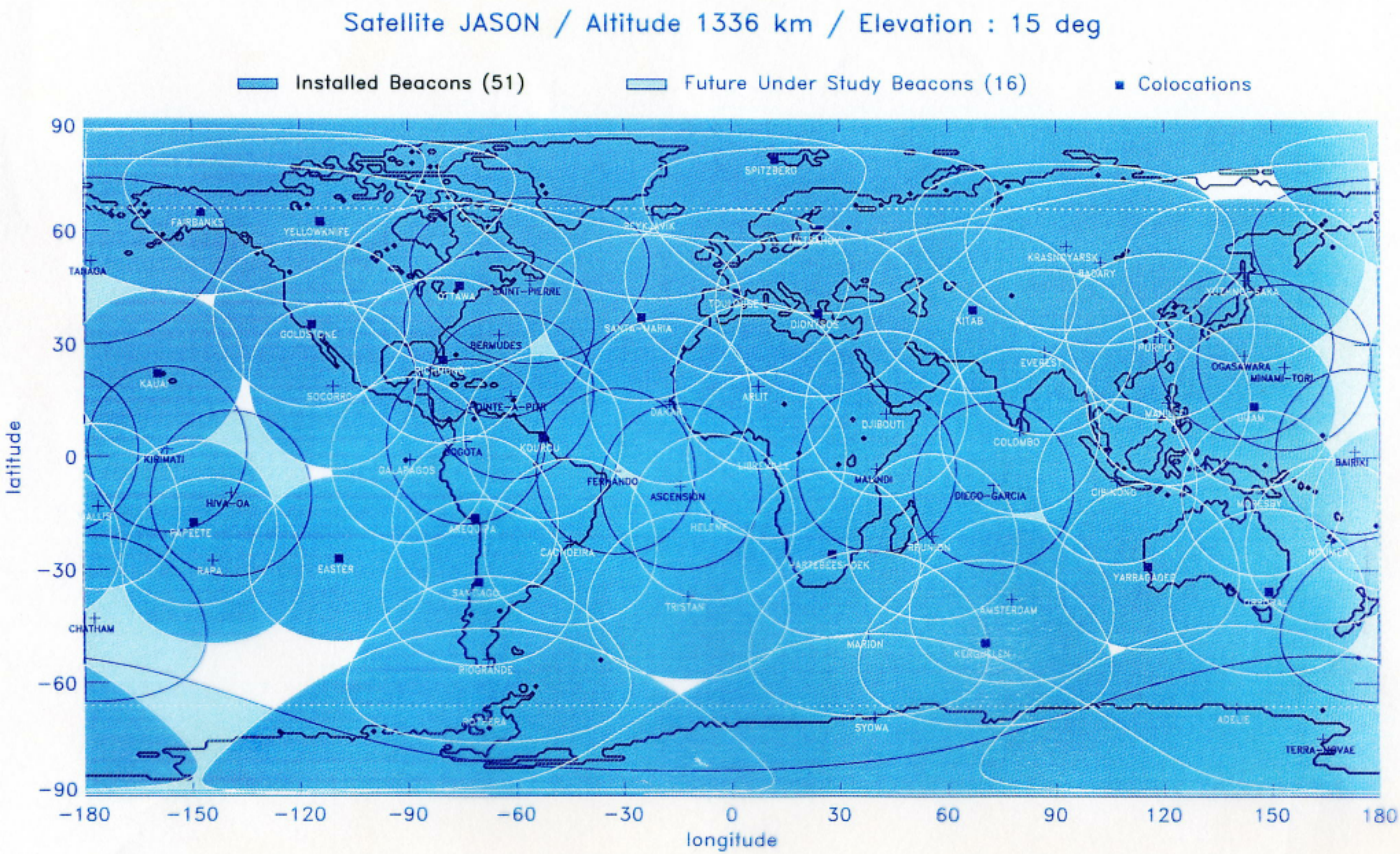
(used for reference system, gravity field, precise positioning)



GPS

Geodetic tracking techniques

Dense: terrestrial network (ex.: DORIS) or Sat-to-sat tracking (GPS)

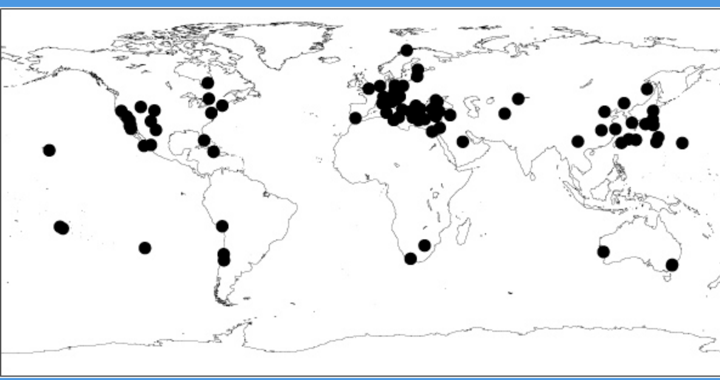


Geodetic tracking techniques

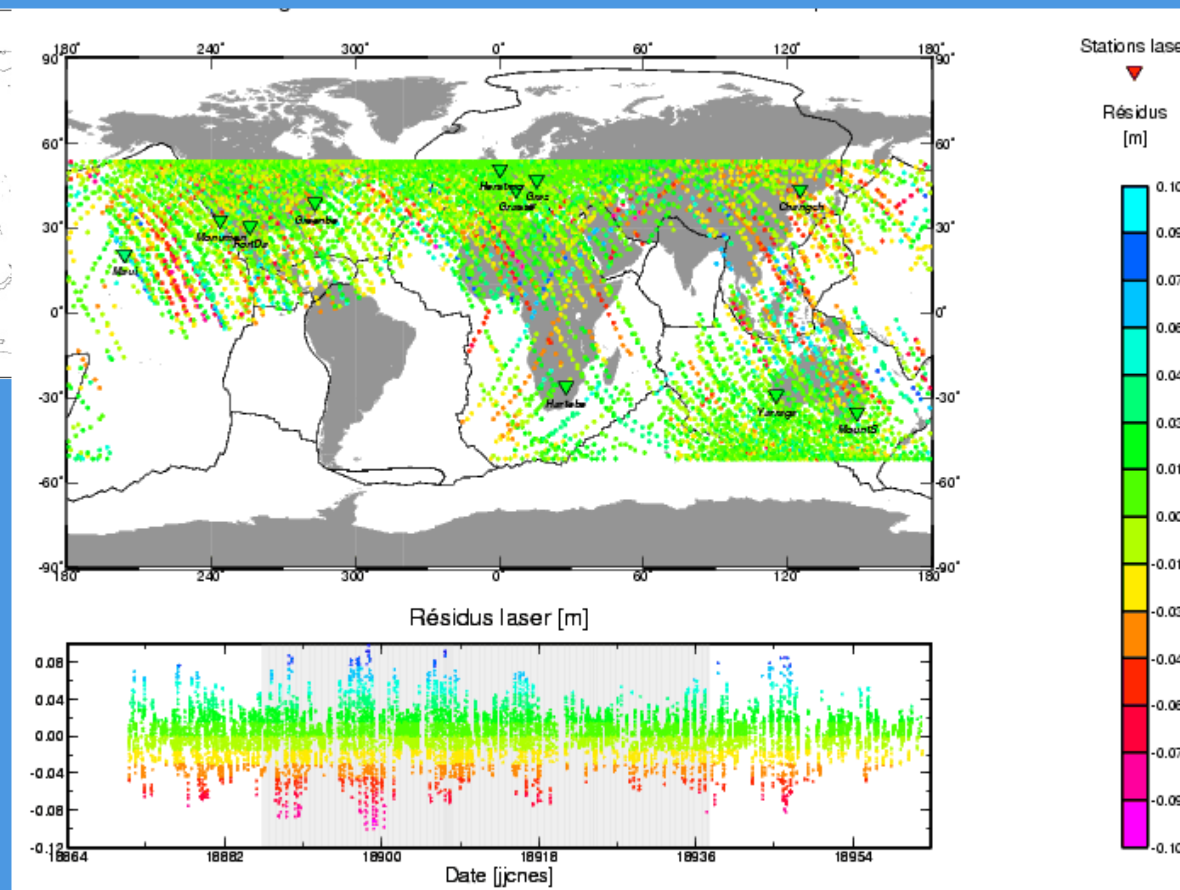
Satellite Laser Ranging (SLR): since the 60s

Simple equation: $$\rho_{station-satellite} = \frac{1}{2} c T_{aller-retour} + \Delta\rho\,(corrections)$$

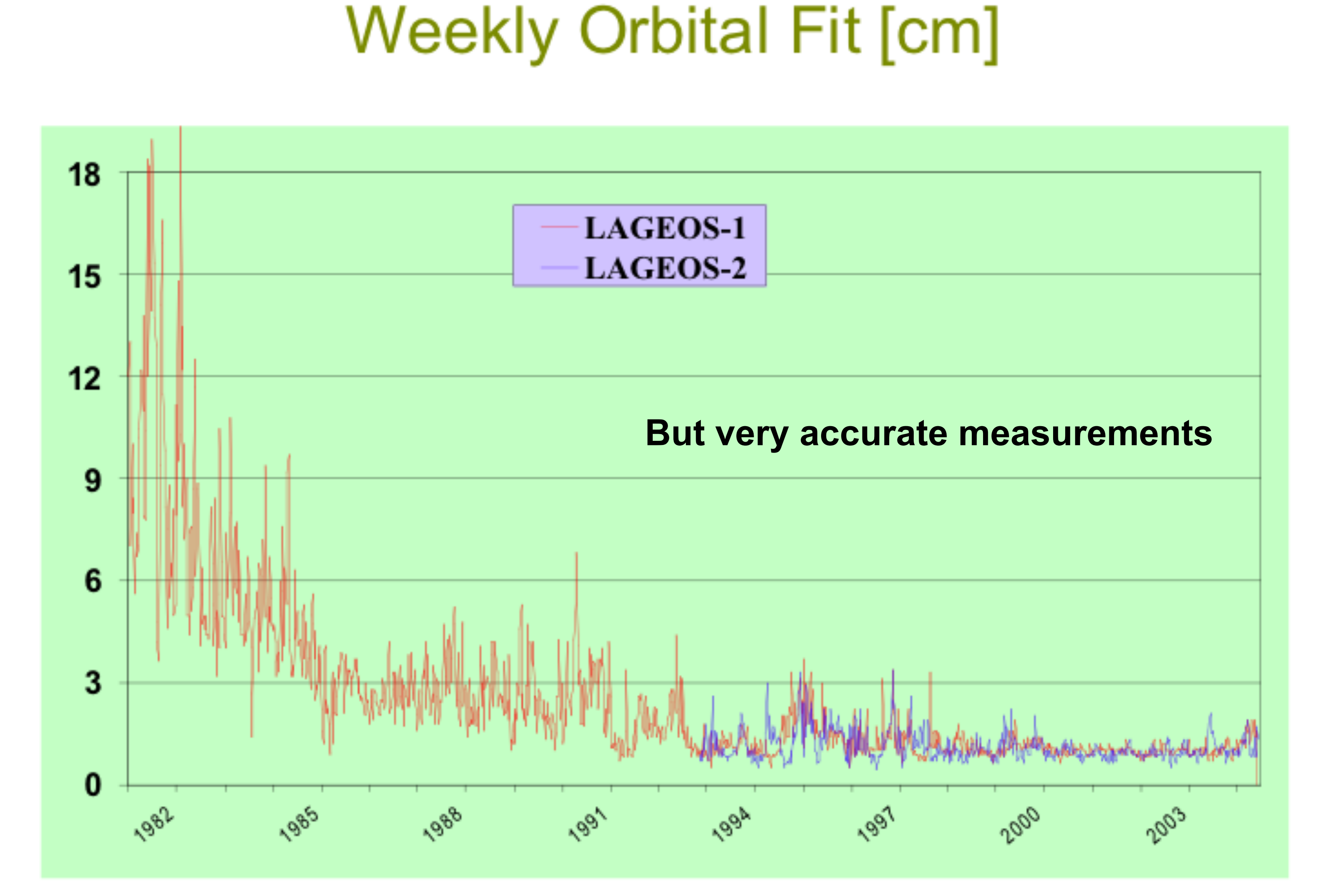
SLR tracks (100 days of LAGEOS-2)



SLR station network: sparse



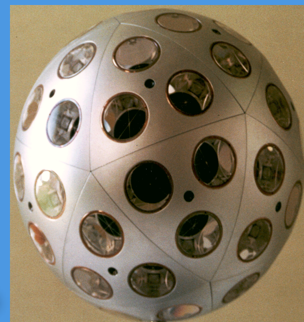
Geodetic tracking techniques



But very accurate measurements

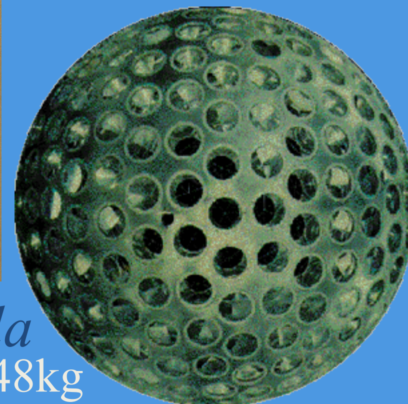
Geodetic satellites

Diadème



Starlette, Stella

Ø=24cm, 60CCR, 48kg



LAGEOS

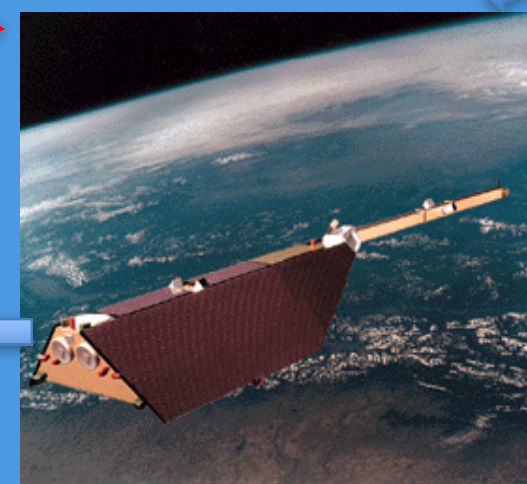
Ø=60cm, 426CCR, 410kg

TOPEX/Poseidon

DORIS+SLR

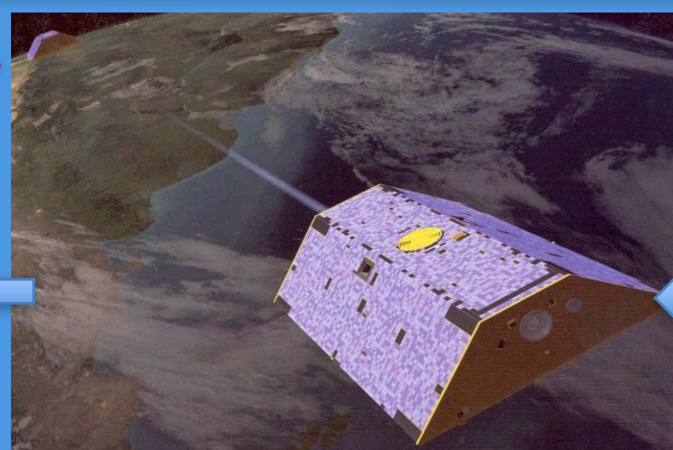
Mean densities

High-res densities



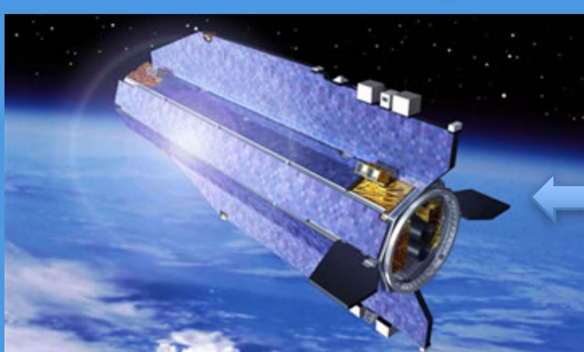
CHAMP

GPS+SLR



GRACE

KBR, GPS+SLR



GOCE

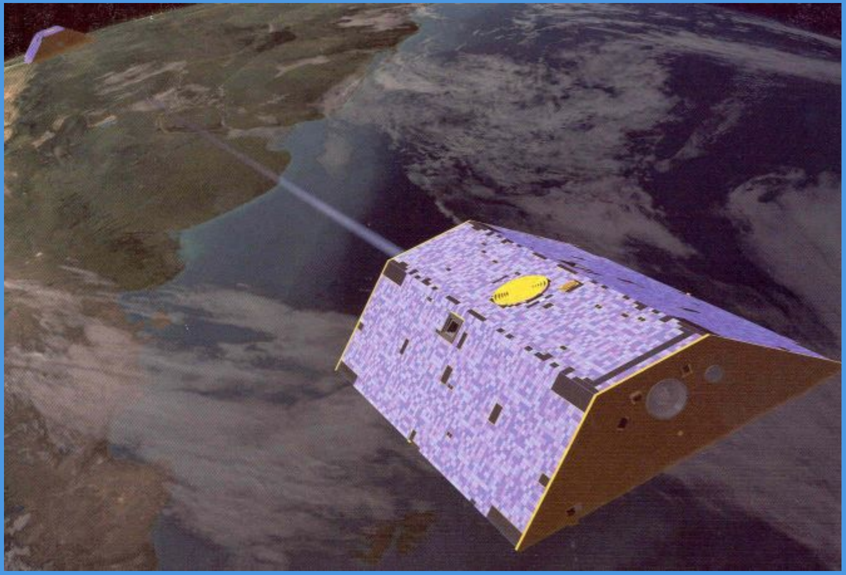
SGG, GPS+SLR

Earth's gravitational attraction

The acceleration is derived from the earth gravitational potential modeled using a development in spherical harmonics:

$$\overline{A} = \overline{\text{grad}}\, U \quad , \quad U = \frac{GM}{a_e} \sum_{l=o}^{L} \sum_{m=o}^{l} \left(\frac{a_e}{r} \right)^{l+1} \overline{P}_{lm}(\sin\varphi)\left(\overline{C}_{lm} \cos m\lambda + \overline{S}_{lm} \sin m\lambda\right)$$

Gravity models presently are, thanks to the GRACE and GOCE missions, very accurate and do not significantly contribute to orbit error.



Atmospheric drag

Atmospheric drag acceleration:

$$a_{drag} = -\tfrac{1}{2} C_D \frac{A}{m} \rho v^2$$

v = satellite speed with respect to co-rotating atmosphere (*orbit*)

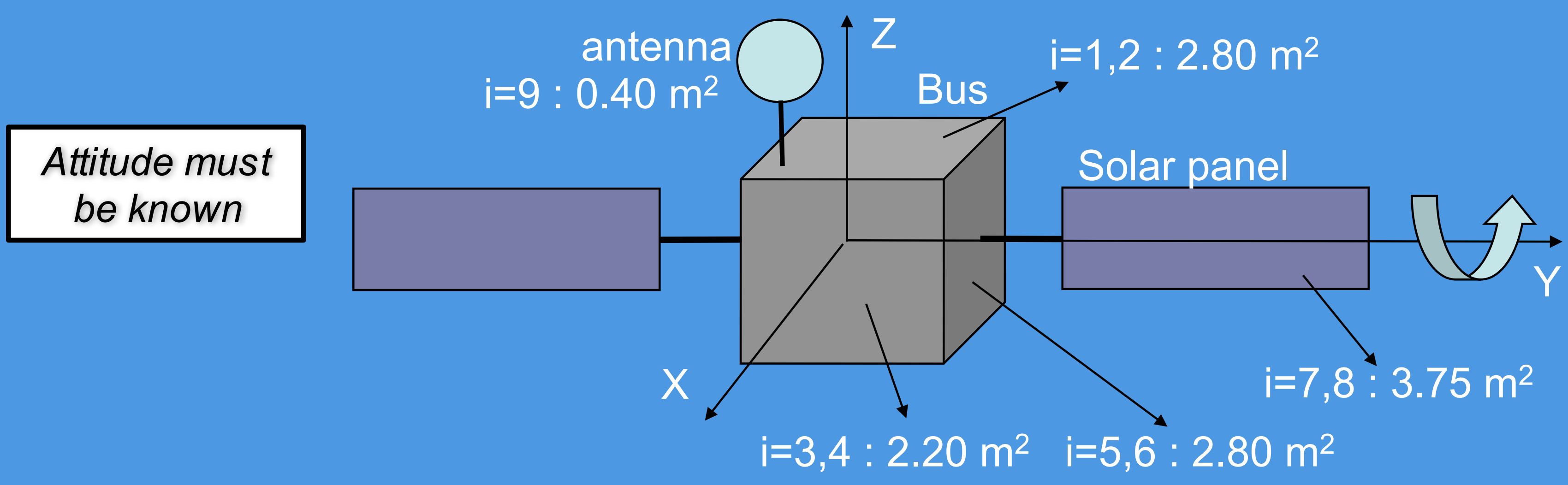
A = satellite surface perpendicular to speed, or ram area *(model)*

m = satellite mass *(housekeeping)*

C_D = aerodynamic coefficient (*model*)

ρ = thermosphere density (*model*)

Simple satellite model (macromodel). Example below: « box-and-wing »



Atmospheric drag

: density

$\bar{v}_r$: relative speed with respect to co-rotating atmosphere and winds

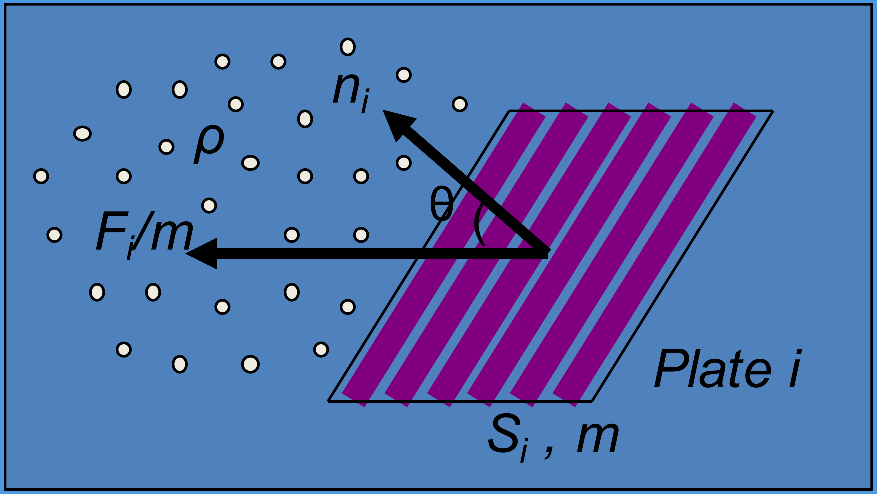
$$\bar{v}_r = \dot{\bar{r}} + \bar{r} \cdot \quad \bar{v}_a : \begin{matrix} \dot{x} + & y & v_x \\ \dot{y} & x & v_y \\ \dot{z} & v_z & \end{matrix}$$

: earth rotation, ω = 7.292115 10-5 rd/s

$\bar{n}_i$: normal vector to plate i

C_{D_i} : drag coefficient

C_{L_i} : lift coefficient



Drag and lift for all plates i:

$$\bar{C}_D = \ _i \, C_{D_i} \frac{S_i}{m} \frac{\bar{v}_r . \bar{n}_i}{v_r^2} . \bar{v}_r$$

$$\bar{C}_L = \ _i \, C_{L_i} \frac{S_i}{m} \frac{\bar{v}_r \cdot \bar{n}_i}{v_r^2} \cdot \bar{v}_r$$

$$\frac{\bar{F}}{m} = \frac{1}{2} \quad v_r^2 \left(\bar{C}_D + \bar{C}_L \right)$$

Atmospheric drag ***(with input from Eelco Doornbos)***

Satellite aerodynamics

Study of forces (and moments) on satellites due to the interaction of atmospheric gas particles with the satellite outer surfaces.

Conservation laws of mass, momentum and energy apply.

Continuum assumption does not hold: mean free path length is much larger than characteristic length of the satellite (Kn>>1).

Therefore, analytical (Cook, Sentman) or statistical (TPMC) methods must be used.

Free molecular flow for low Earth orbiting satellites

Characterized by extremely low density, high orbital velocity; Impossible to simulate these conditions in wind tunnels.

No satellite mission has ever made simultaneous direct observations of all the variables required to fully characterize satellite aerodynamic interaction (composition, temperature & acceleration – *but scale and bias problems*).

Atmospheric drag

Exact analytical expressions can be derived for flat plates and other basic shapes (spheres, cylinders, etc.).

Test Particle Monte Carlo (TPMC) method required for concave shapes. Monte Carlo methods are the only way to accurately account for multiple reflections.

Self Shadowing may play an important role (depending on shape and attitude)

ERWAN MAZARICO PH.D. THESIS

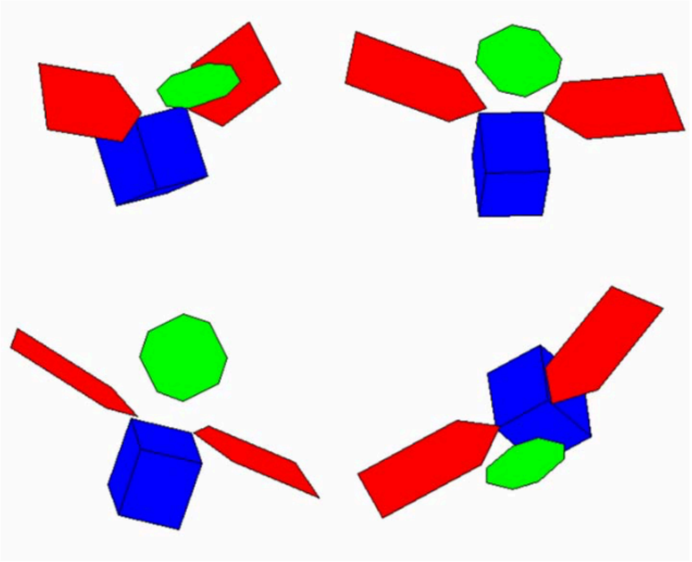


Figure 5.7 MRO spacecraft seen from 4 different view points. The bus is in blue, the solar arrays in red and the high-gain antenna in green.

Atmospheric drag

Drag coefficients can be calculated in two ways:

1. Based on tracking data and a density model; the drag coefficient estimate includes contribution of density model error

→ *Not possible when using tracking data to infer total density*

2. Computation of drag coefficient with models (physical principles)

Atmospheric drag

INTERNATIONAL STANDARD

ISO 27852

First edition
2011-07-15

Space systems — Estimation of orbit lifetime

Systèmes spatiaux — Estimation de la durée de vie en orbite

6.2 Estimating drag coefficient

A reasonable value of the dimensionless drag coefficient, C_D, is 2.2 for a typical spacecraft. However, C_D depends on the shape of the satellite and the way air molecules collide with it. However, for certain geometric configurations such as spheres, cylinders and cones, the value of C_D can be evaluated more precisely than previously noted provided something is known about the flow regime and reference area[4]. The analyst shall consider C_D variations based on satellite shape. However, for long-duration orbit lifetime estimations, C_D variation as a function of orbit altitude[4] may safely be ignored, since the orbit lifetime percent error is quite small due to averaging effects about the adopted 2.2 value.

Atmospheric drag

Smithsonian Astrophysical Observatory Special Report 171, 1965

DENSITIES AND TEMPERATURES FROM THE

ATMOSPHERIC DRAG ON SIX ARTIFICIAL SATELLITES[1]

by

Luigi G. Jacchia[2] and Jack Slowey[3]

Method of computation

The methods used to determine orbital accelerations and to compute densities were essentially the same as have been described before (Jacchia and Slowey, 1963a) and need not be repeated in detail. The contribution to the accelerations due to solar-radiation pressure was computed using Kozai's formulation of the effect (Kozai, 1959) assuming specular reflection from a spherical surface and a value of the solar constant of 2.00 cal cm^{-2} min^{-1}. The densities were computed by numerical integration of Sterne's integral--in the form including atmospheric rotation (Sterne, 1958, 1959)--assuming the drag coefficient to be 2.2 for all of the satellites.

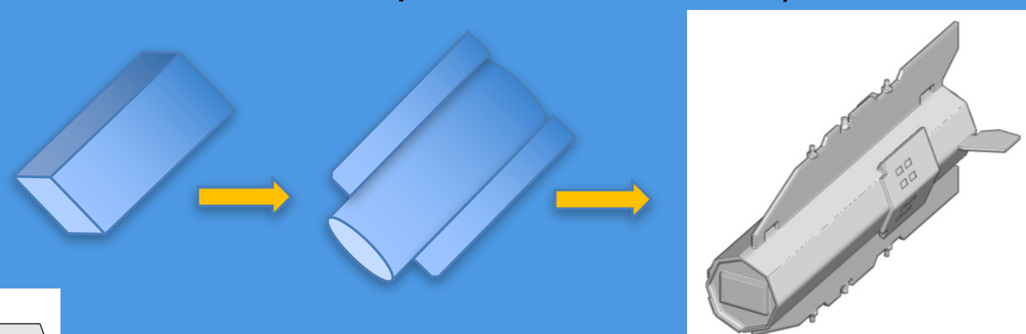
Atmospheric drag

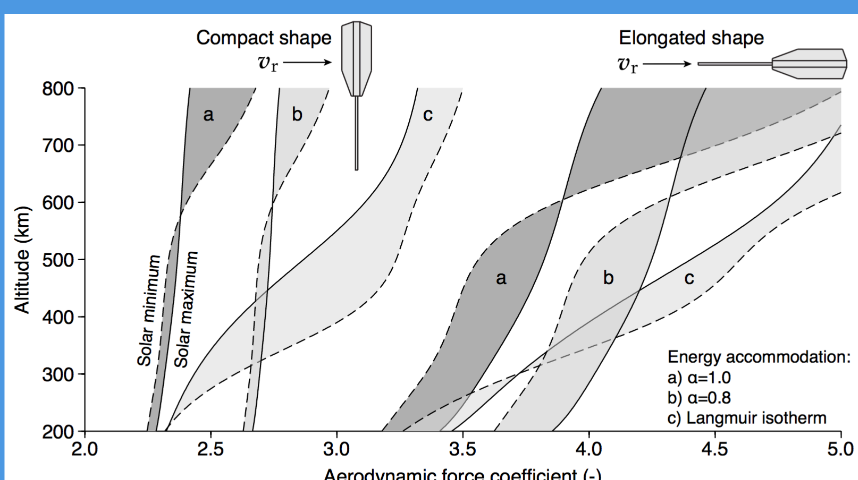
The present situation: rarefied aerodynamics theory is complete and computational methods are well developed.

<u>However, there is no standard, not even consensus within the community.</u>

This leads to differences in the drag calculation due to:

- Level of approximation of the satellite model (equivalent sphere, small number of panels, high-res, undocumented changes, mass evolution,…)
- Choice of drag coefficient (constant, model, assumption within model)





<u>Examples of differences (within model):</u>

$$s = \frac{v_{sat}}{v_{molecular}};\ v_{mol} = \sqrt{RT/_M} \text{ or } v_{mol,i} = \sqrt{RT/_{m_i}}$$

$$\alpha = \frac{3.6\mu \text{ or } 4\mu}{(1+\mu)^2};\ \mu = \frac{mass\ incident\ gas}{mass\ surface\ atom} = \frac{M \text{ or } m_i}{16 \text{ or } ?}$$

Atmospheric drag

Consequences of inconsistent use of drag coefficients:

- In the past, C_d=2.2 has been used to derive density (Jacchia)
- The densities inferred from accelerometers (CHAMP, GRACE, GOCE) have been obtained with several drag coefficient models, leading to large differences (20-30%)
- Thermosphere models are fitted to data of different origins – and underlying C_d models – which required scaling of datasets for model consistency

A New Empirical Thermospheric Density Model JB2008 Using New Solar and Geomagnetic Indices

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for the CHAMP and GRACE data. The displayed CHAMP density ratios are orbit averaged values / yearly average, and then multiplied by 1.17 to adjust to the HASDM values. The 17% factor is based on averaging the CHAMP/HASDM ratios over the 2001-2005 time period. A factor of 0.74 was obtained for the GRACE/HASDM ratios based on all data from 2002 through 2005. The HASDM values plus other

Validation of GOCE densities and evaluation of thermosphere models

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Abstract

Atmospheric densities from ESA's GOCE satellite at a mean altitude of 270 km are validated by comparison with predictions from the near real time model HASDM along the GOCE orbit in the time frame 1 November 2009 through 31 May 2012. Except for a scale factor of 1.29, which is due to different aerodynamic models being used in HASDM and GOCE, the agreement is at the 3% (standard deviation) level when comparing daily averages. The models NRLMSISE-00, JB2008 and DTM2012 are compared with the GOCE data. They match at the 10% level, but significant latitude-dependent errors as well as errors with semiannual periodicity are detected. Using the 0.1 Hz sampled data leads to much larger differences locally, and this dataset can be used presently to analyze variations down to scales as small as 150 km.

Atmospheric drag

<u>Orbit computation with ρ calculated with *modelA*, for model evaluation:</u>

C_d=2: *a*=1e-8 m/s^2 estimated drag scale factor: 1

C_d=4: *a*=2e-8 m/s^2 estimated drag scale factor: 0.5 ➡ '*modelA biased*'

<u>Density inferred from accelerometer/POD:</u>

C_d=2: ρ=2e-16 kg/m^3 assimilated in *modelA*

➡ ρ *modelA* = 2x ro *modelB*

C_d=4: ρ=1e-16 kg/m^3 assimilated in *modelB*
(models assimilate data from many sources; C_d often unknown)

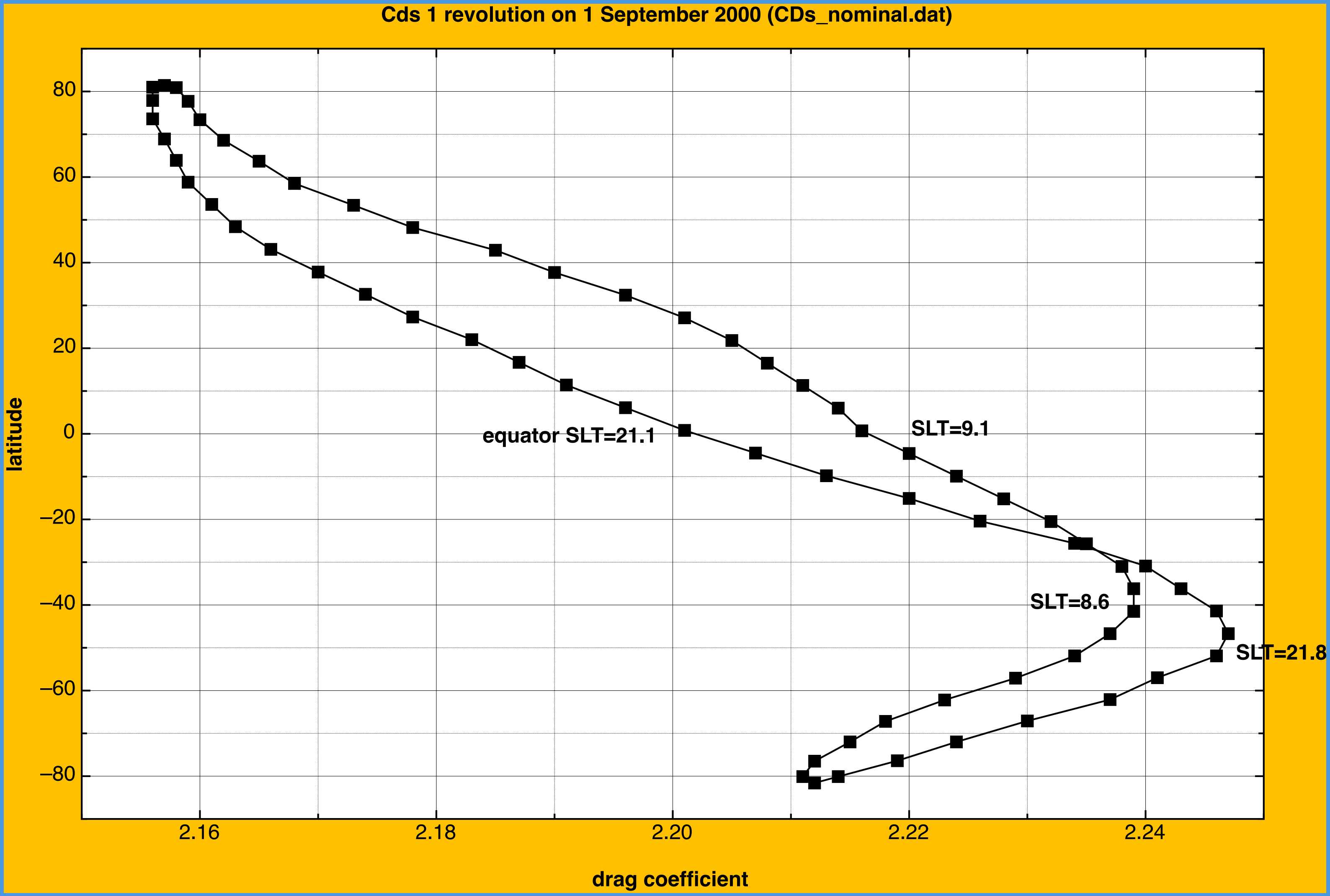
<u>Orbit computation with ρ calculated with *modelA or modelB, C_d=2 or C_d=4*:</u>
(assuming identical satellite macro-model…)

C_d=2, *modelA*	*a*=4e-8 m/s^2	estimated drag scale factor: 1
C_d=4, *modelA*	*a*=8e-8 m/s^2	estimated drag scale factor: 0.5
C_d=2, *modelB*	*a*=2e-8 m/s^2	estimated drag scale factor: 2
C_d=4, *modelB*	*a*=4e-8 m/s^2	estimated drag scale factor: 1

Atmospheric drag

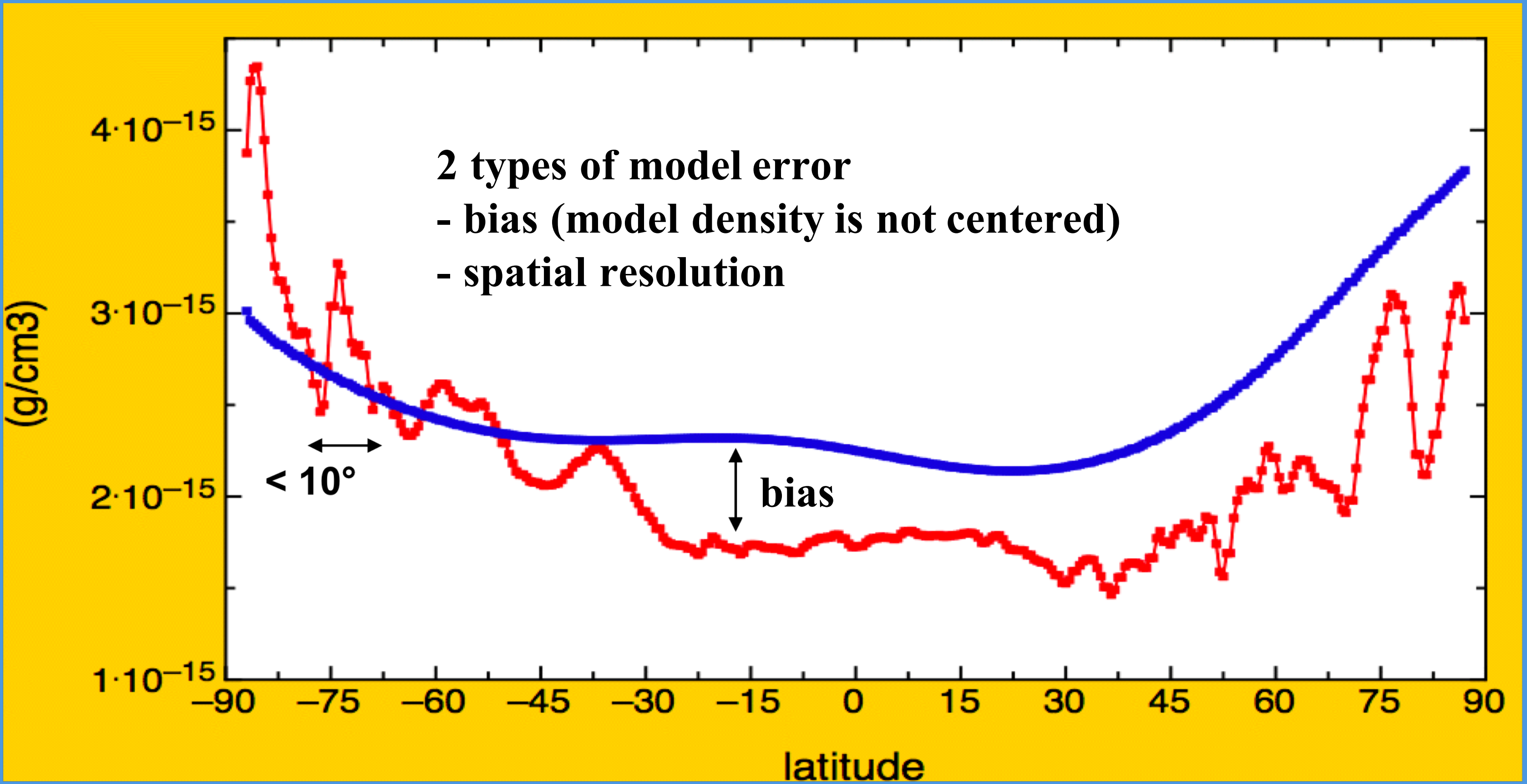
And a final complication: C_d is a function of latitude and local time

the variations map directly into the inferred densities



Atmospheric drag

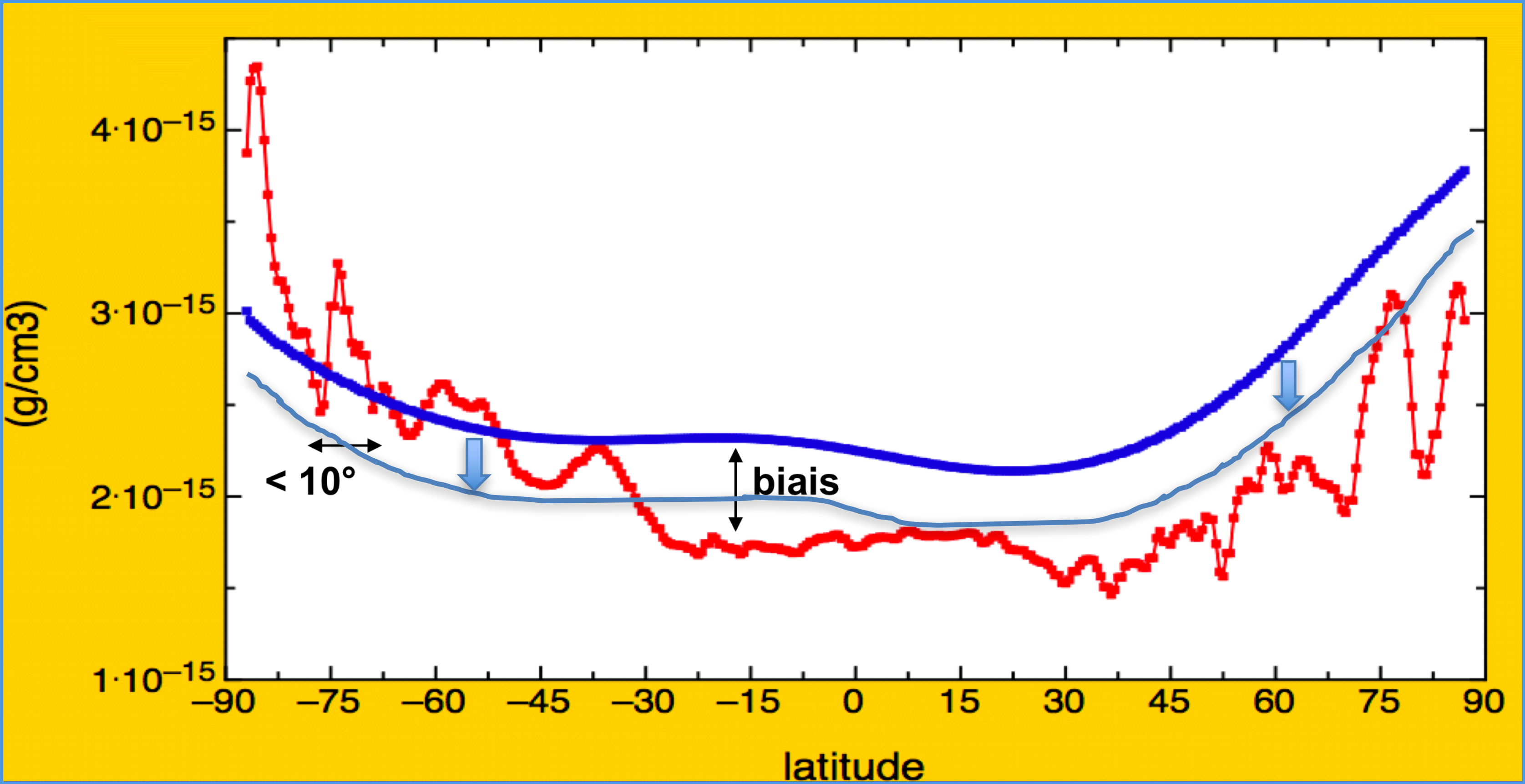
Observed versus modeled density (NB: accelerations look similar)



Atmospheric drag

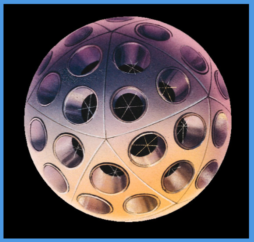
Thanks to tracking data we can correct for model bias:

Estimation of a drag scale factor



Atmospheric drag

In POD, remaining model error must be as small as possible: estimation of many drag scale factors (NB: tracking data allowing)

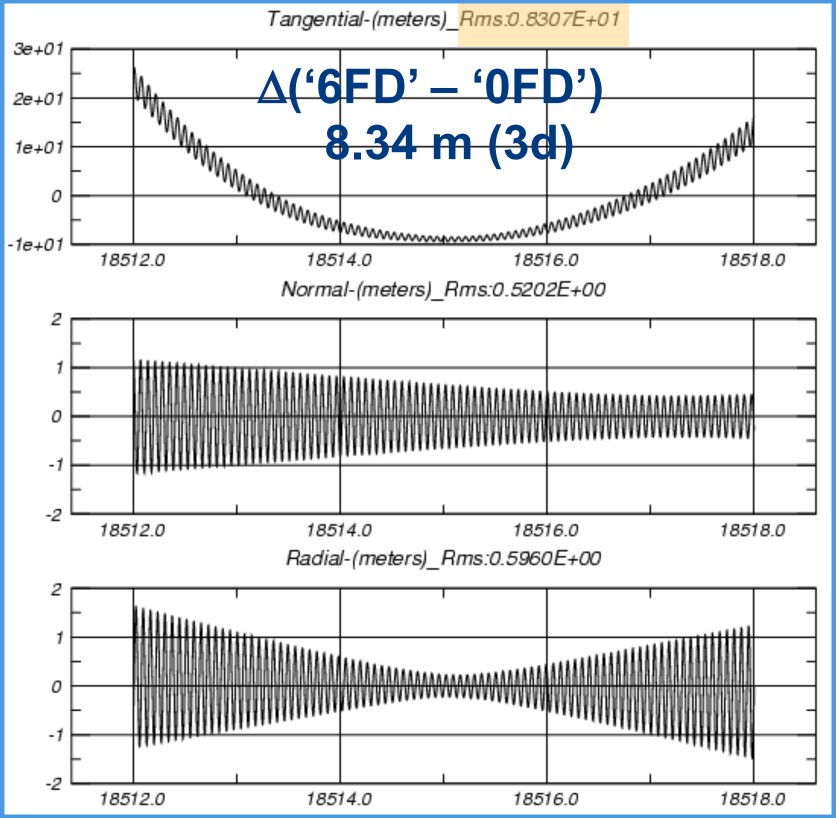


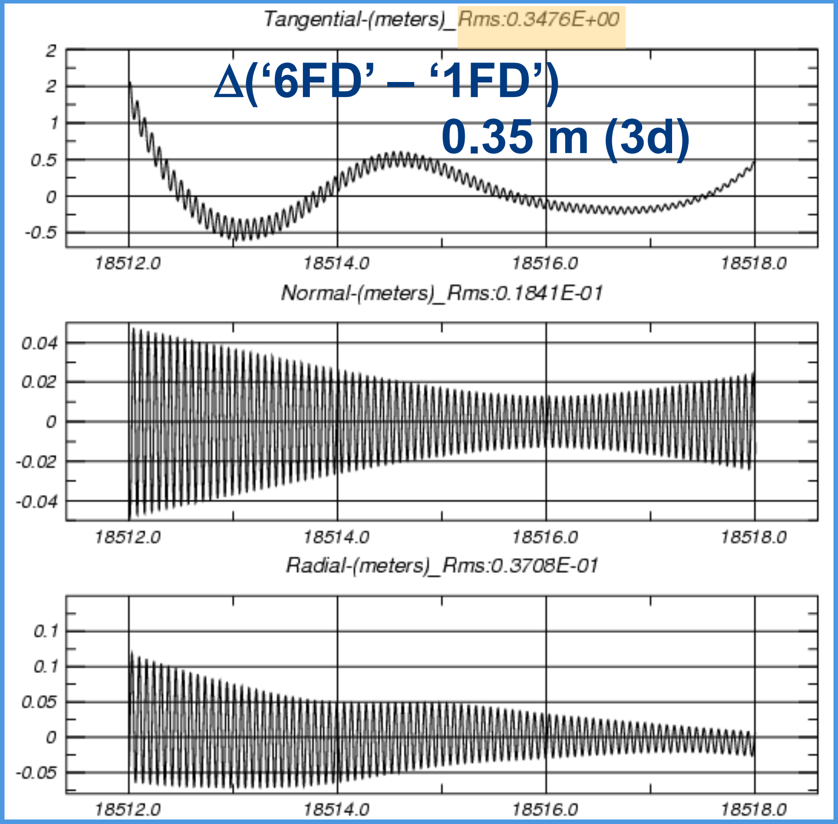
<u>Stella 6-day orbit (altitude: 820 km), adjusted to SLR data.</u>

RMS SLR residuals, model without FD: 3.93 m

RMS SLR residuals, model with 1 FD: 0.16 m / **FD=1.25**

RMS SLR residuals, model with 6 FD: 0.04 m / **FD=1.36, 1.30, 1.16, 1.29, 1.27, 1.31**





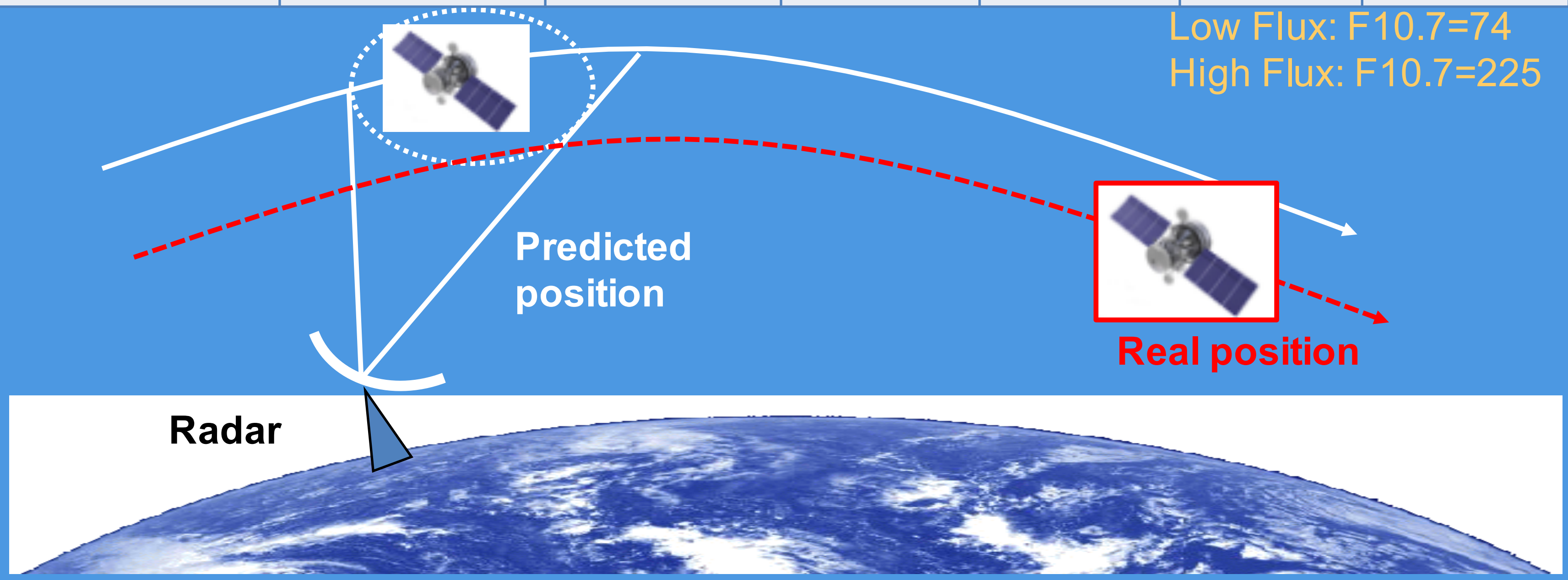
Atmospheric drag

Atmospheric drag computation is not accurate because of:

- Thermosphere model (1-σ) precision of 5-25%
- Aerodynamic coefficient, mass, attitude and macromodel error (5-??%)
- *Solar and geomagnetic activity forecast (days, month, years)*

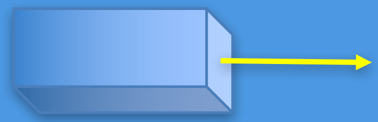
	250 km		550 km		850 km	
	LF	HF	LF	HF	LF	HF
Total effect	89.8 km	414.0 km	146.3 m	4.2 km	15.8 m	195.6 m
* 0.5	45.0 km	209.4 km	73.4 m	2.1 km	7.9 m	98.1 m
Δ models	5.7 km	12.2 km	42.1 m	0.8 km	5.6 m	60.7 m

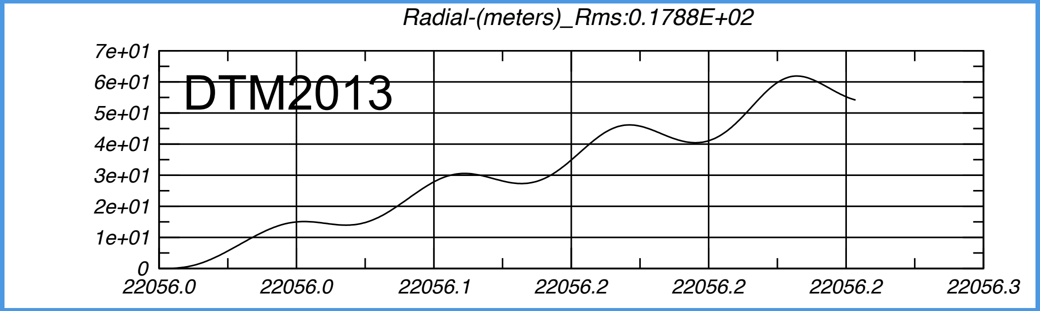
Low Flux: F10.7=74
High Flux: F10.7=225



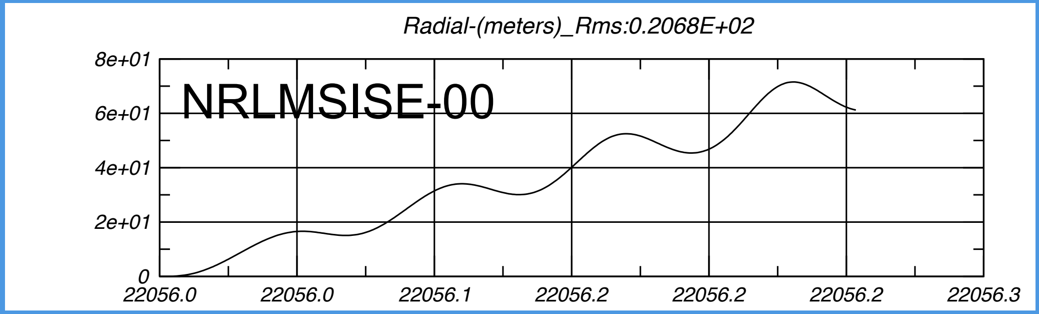
Atmospheric drag

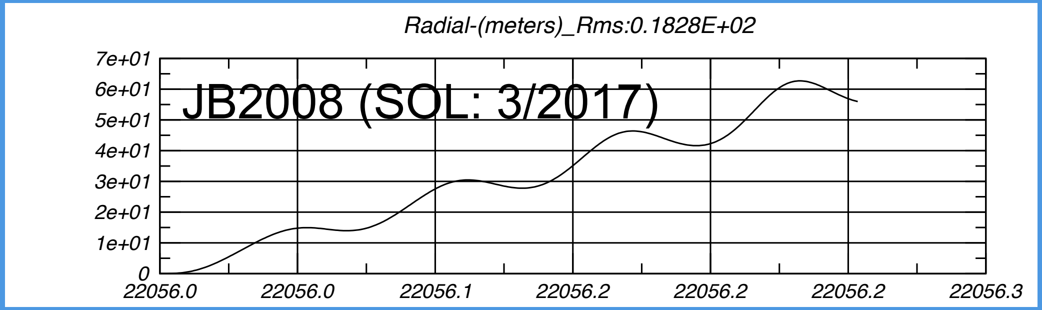
Orbit extrapolations: polar orbit at about 400 km, box model 1m² frontal area (May '10)



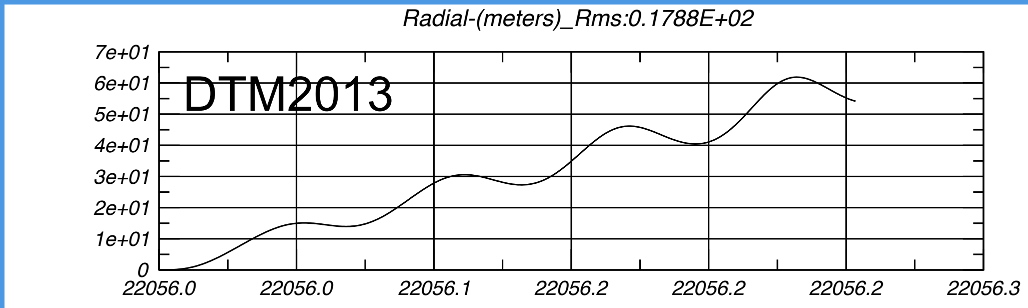


Total drag effect
(6hr extrapolation)

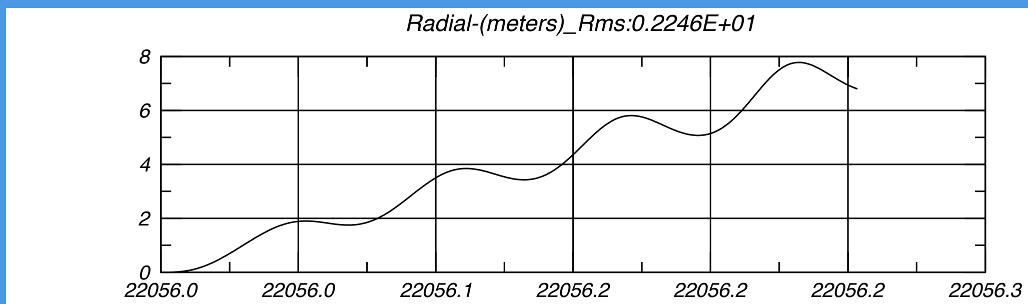




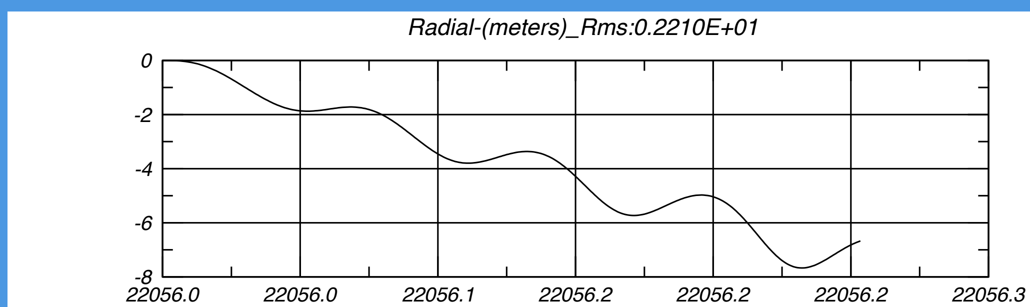
Atmospheric drag



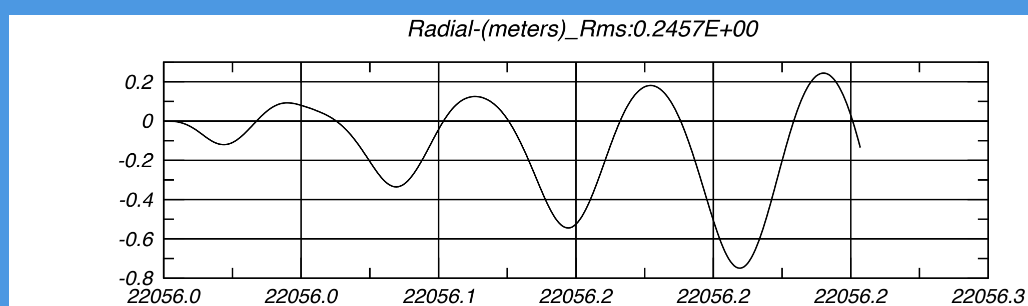
Total effect



Δ area 1.2 vs 1 m^2

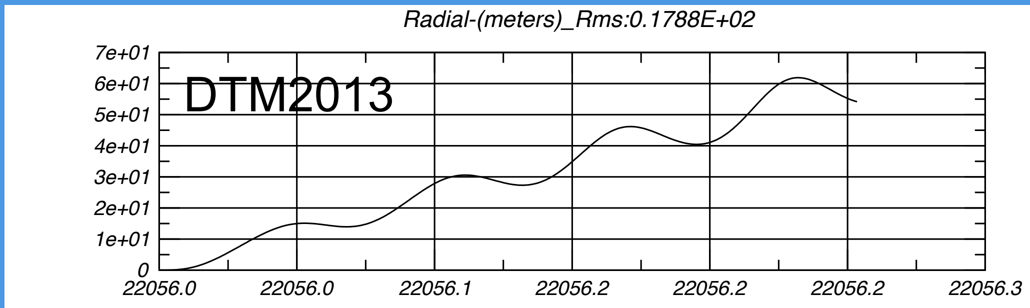


Δ alpha=4.0 vs 3.6

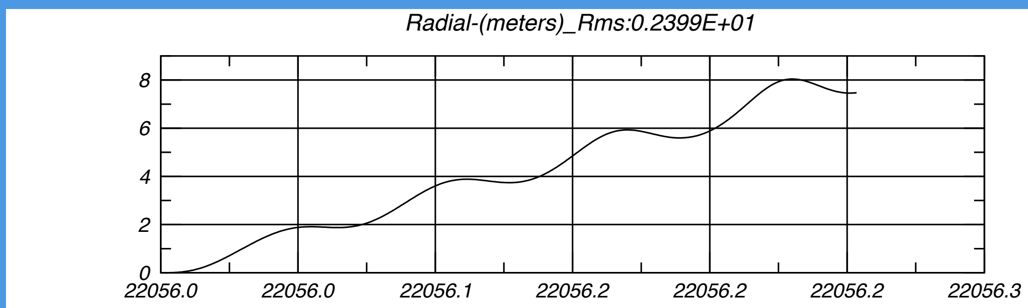


Δ box vs
tuned sphere

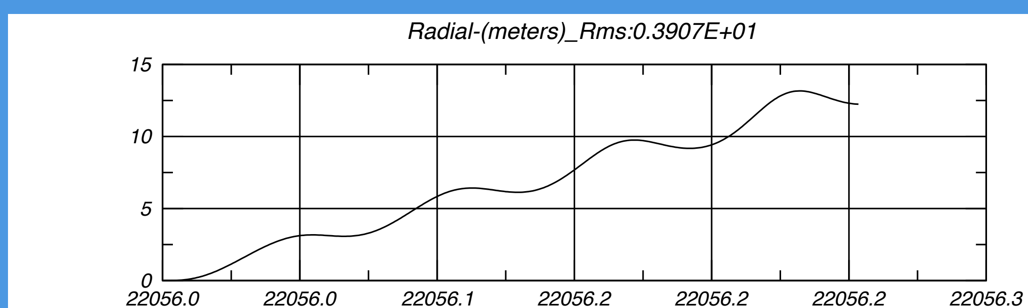
Atmospheric drag



Total effect

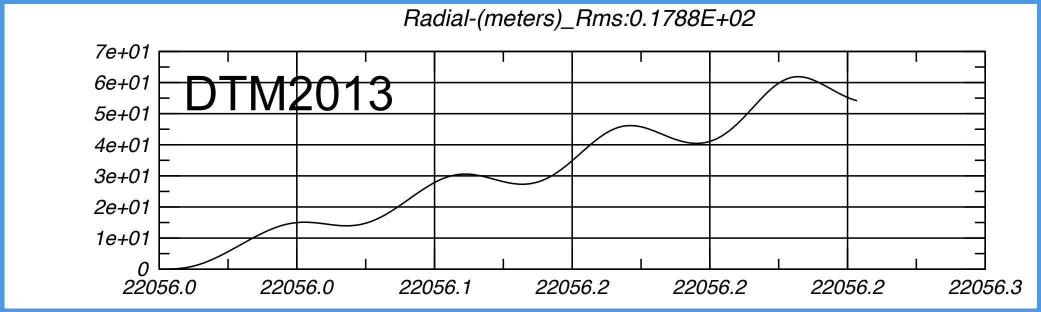


Δ F30 + 10 sfu

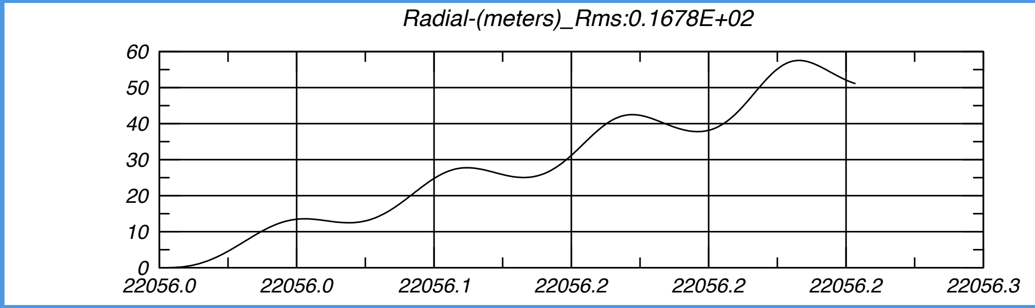


Δ F30 & F30m + 10 sfu

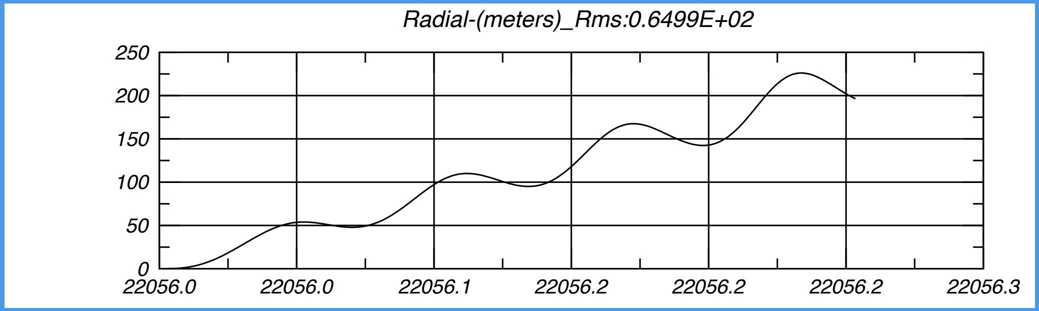
Atmospheric drag



Total effect



Δ Kp=5 vs observed (1-2)



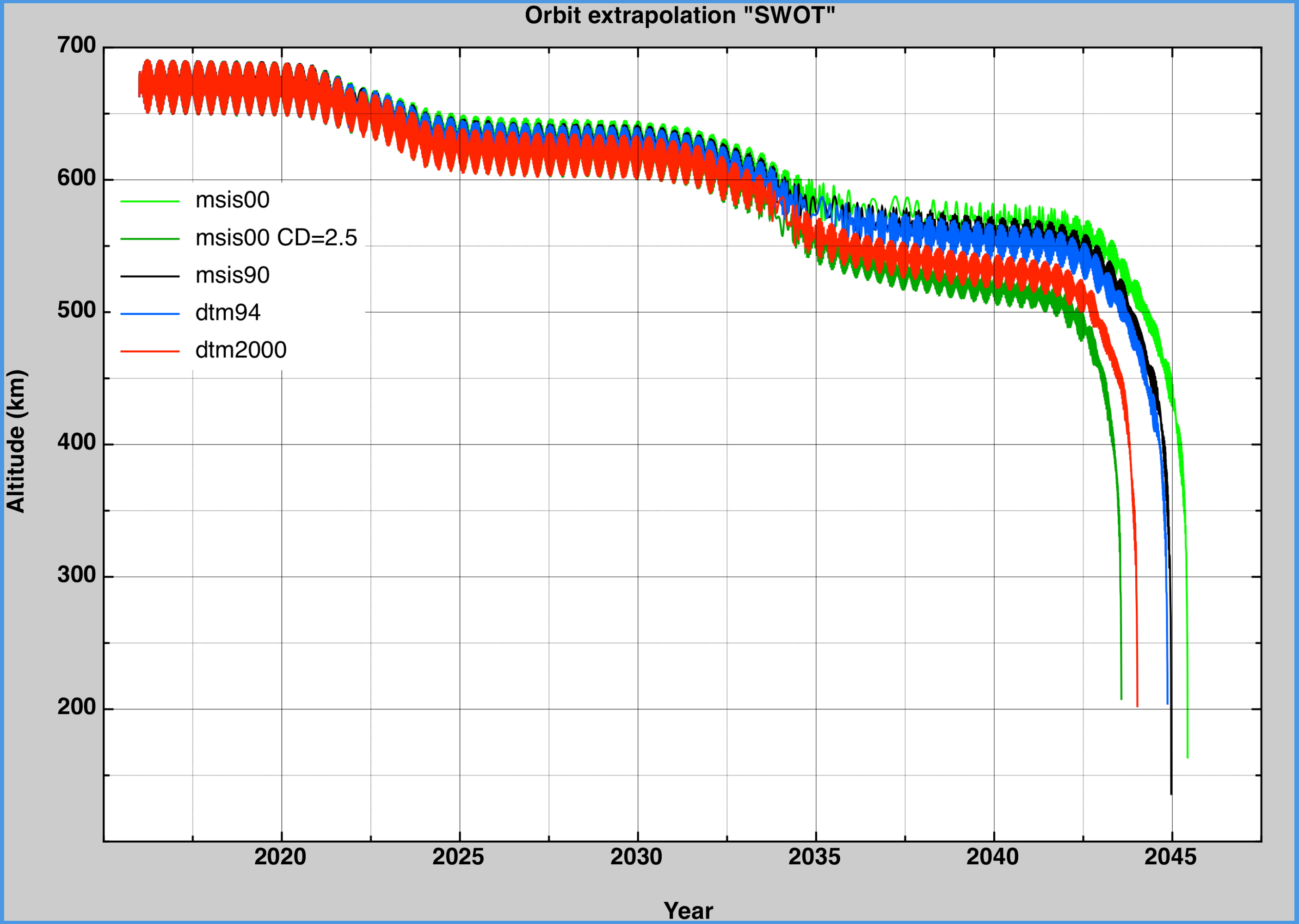
Δ Kp=9

Atmospheric drag – long time scale

Atmospheric drag computation is not accurate because of:

- *Solar and geomagnetic activity* ***forecast*** *(days, month, years, solar cycle)*

Example: 25 year reentry simulations for satellite SWOT



Atmospheric drag – long time scale

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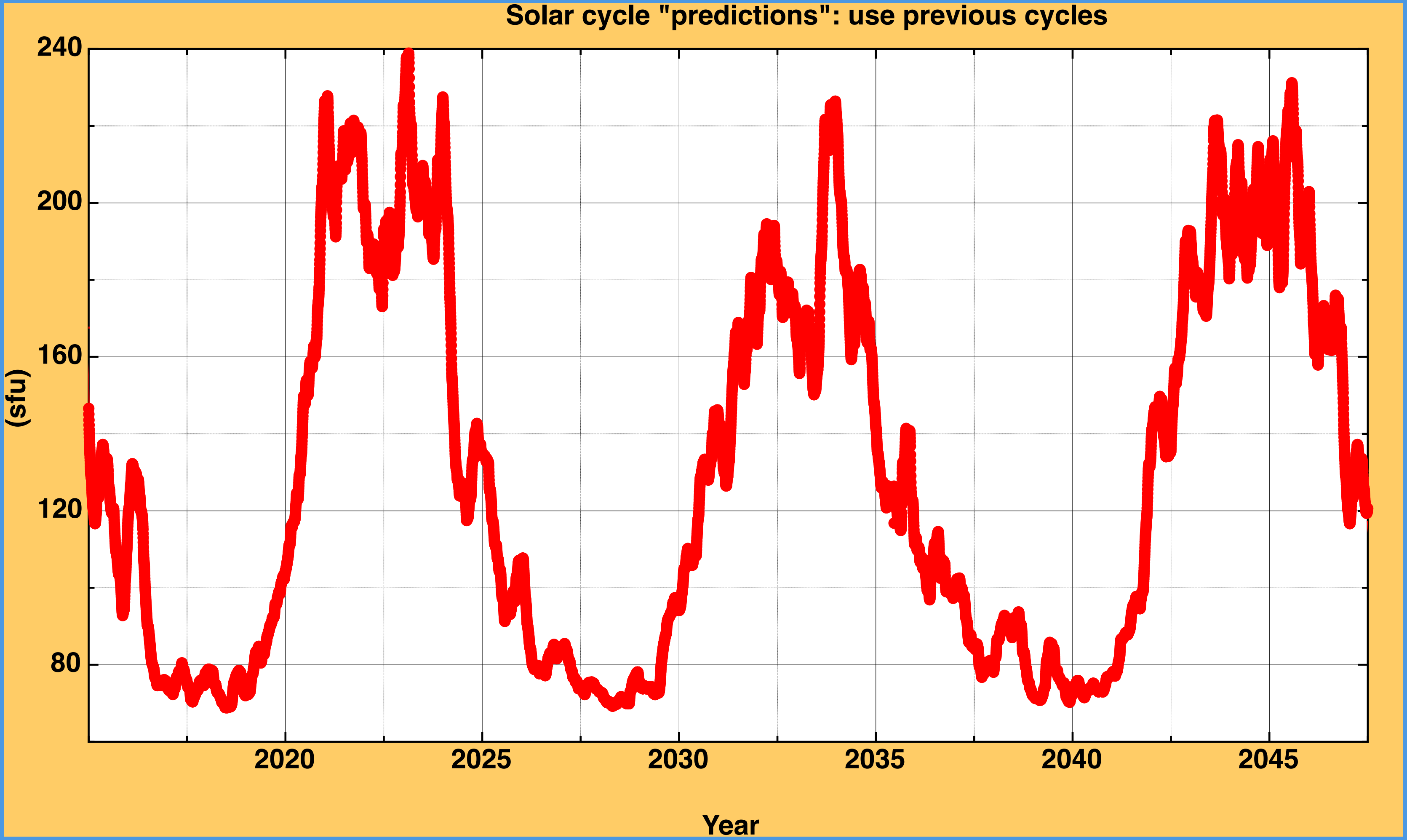
Example: 25 year reentry simulations for satellite SWOT

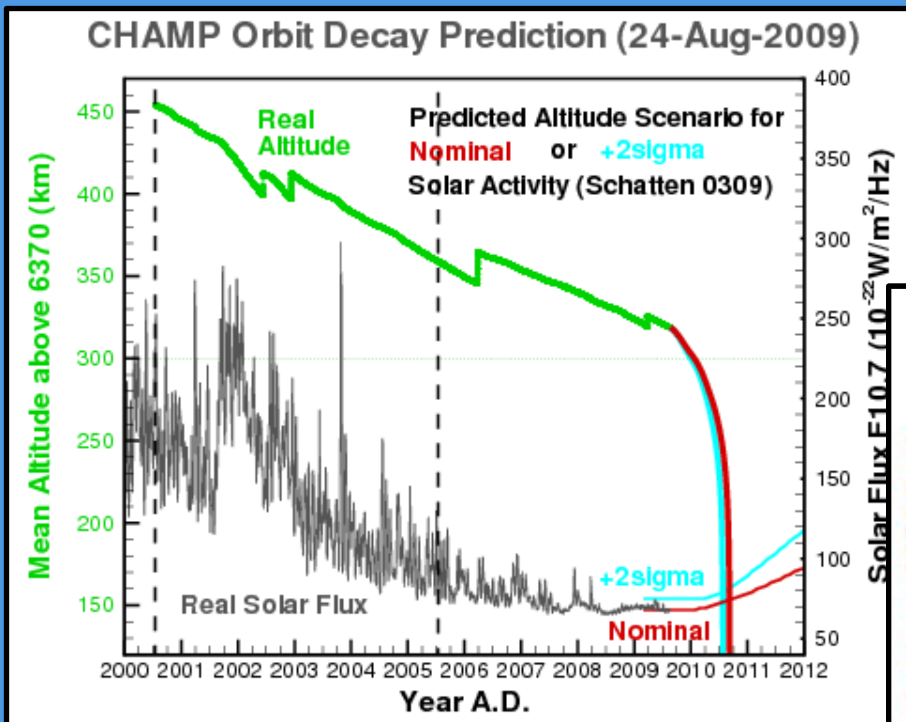


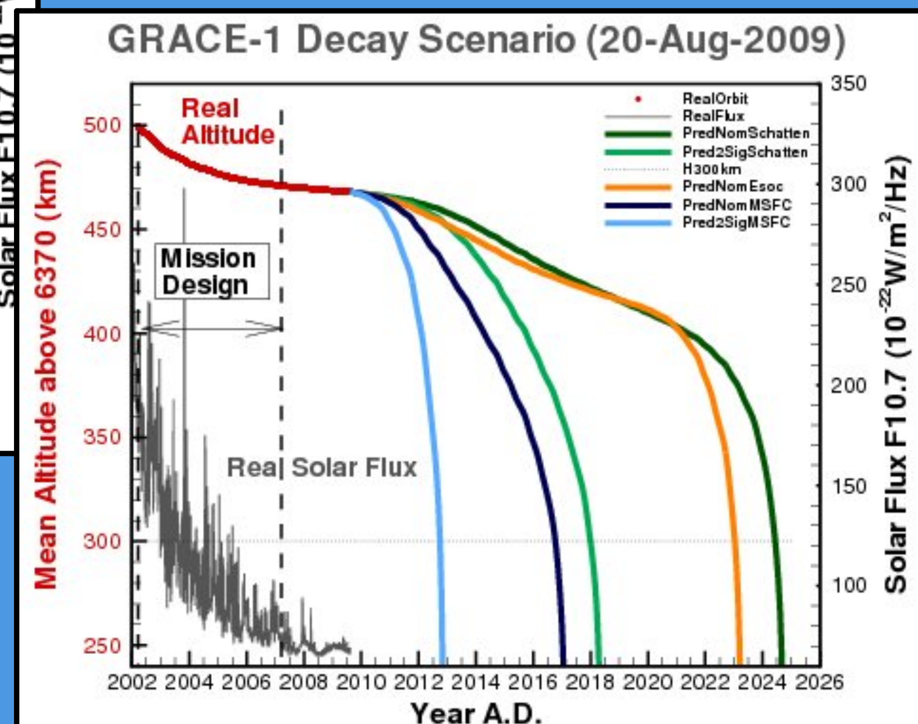
Atmospheric drag – long time scale

Atmospheric drag computation is not accurate because of:

- *Solar and geomagnetic activity <u>forecast</u> (days, month, years, solar cycle)*

Example: CHAMP and GRACE (courtesy GFZ Potsdam)



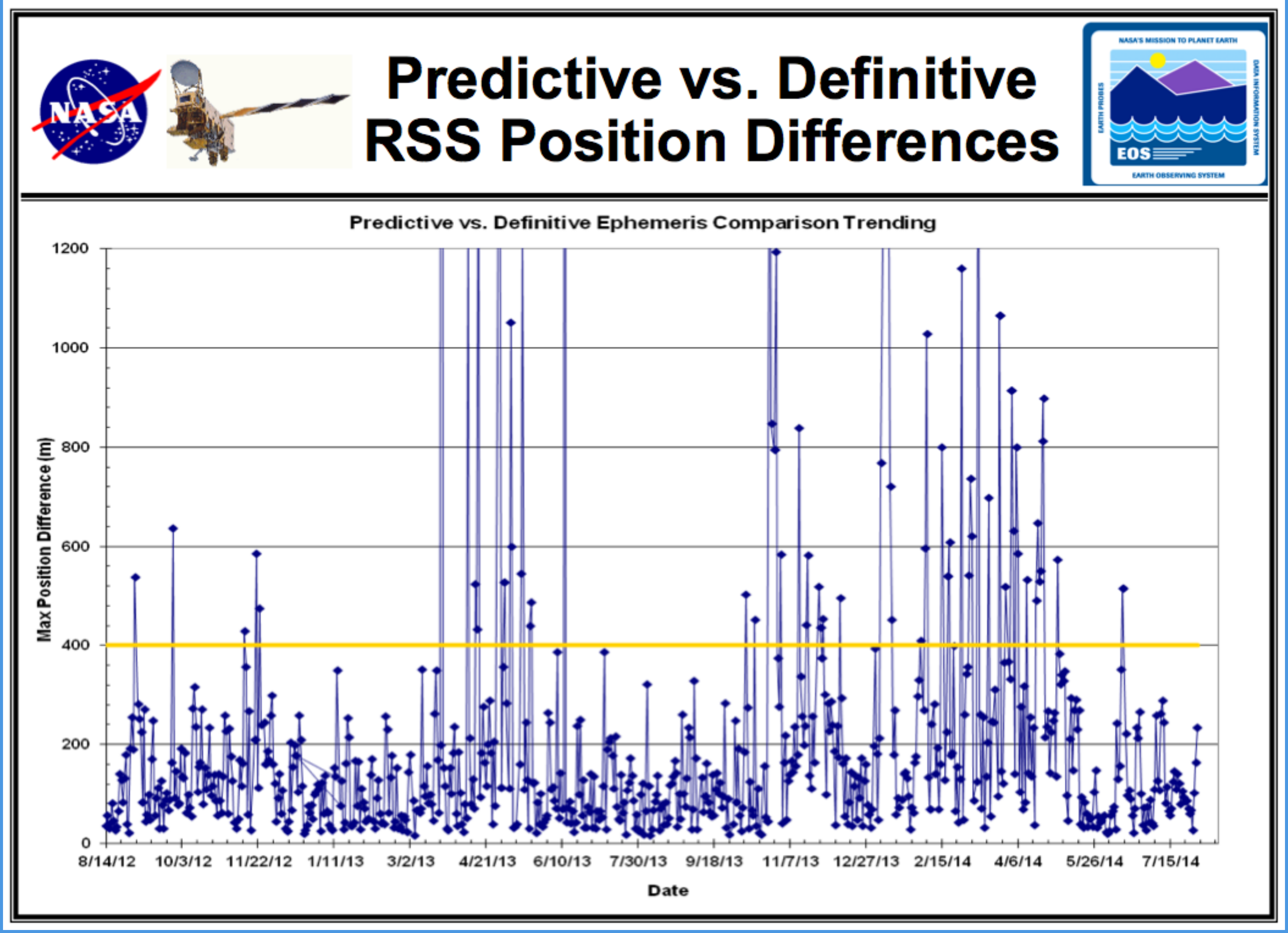


Back up slides

Atmospheric drag – orbit perturbation

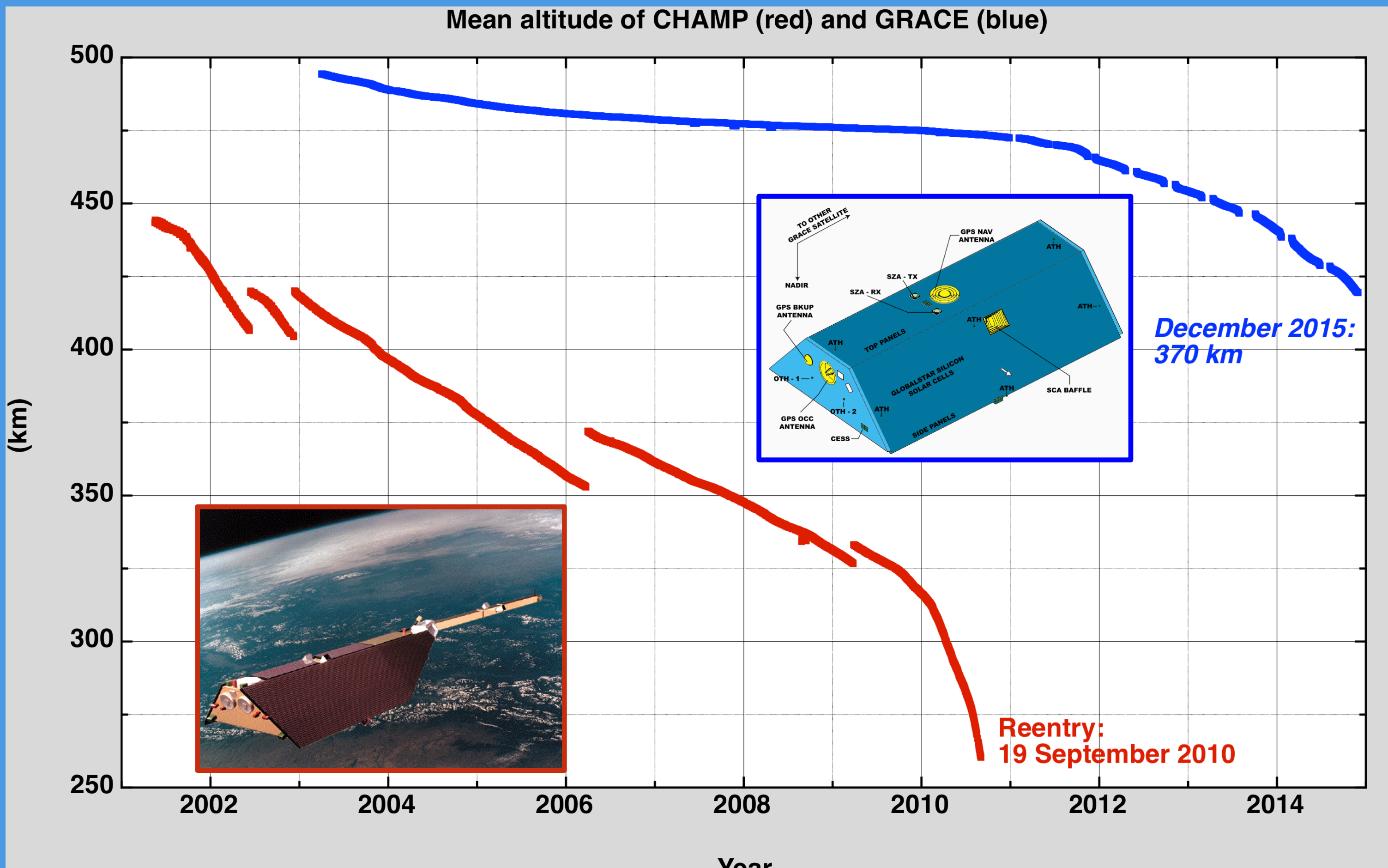
NASA's Aqua satellite at approximately 700 km

Requirement for predicted orbit: error less than 400 m after 32 hours

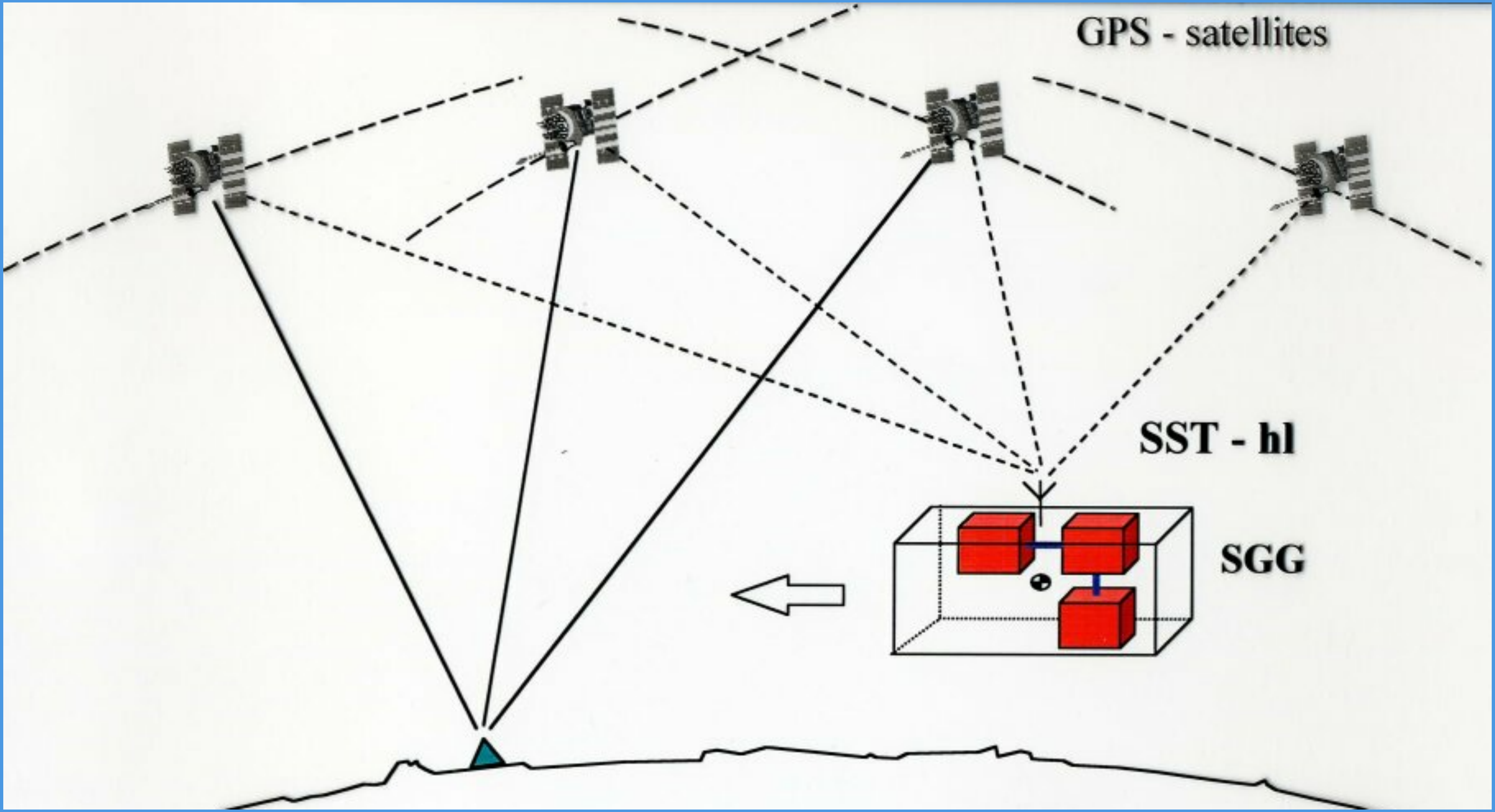


Orbit computation: force model

A satellite/object in Low Earth Orbit loses altitude due to interaction with the neutral air particles (the thermosphere).
Ultimately, it reenters and burns up in the atmosphere.



Gradiométrie, mesurer les gradients de gravité
= SGG (Satellite Gravity Gradiometry)
GOCE: SST-hl + SGG



Modélisation du champ de gravité : atténuation en altitude

Altitude attenuation

400 km : $R/r = 0.941$ $(R/r)^5=0.738$
1000 km: $R/r = 0.864$ $(R/r)^5=0.483$
20000 km: $R/r = 0.242$ $(R/r)^5=0.0008$

L'atténuation est plus faible après différentiation du potentiel T :

$$T = \frac{GM_o}{R} \sum_{n=2}^{\infty} \left(\frac{R}{r}\right)^{n+1} \sum_{m=o}^{n} \left[\Delta\overline{C}_{nm} \cos m\lambda_P + \Delta\overline{S}_{nm} \sin m\lambda_P\right] \overline{P}_{nm}(\cos\theta)$$

$$T_r = \frac{\partial T}{\partial r} = -\gamma \sum_{n=2}^{\infty} (n+1)\left(\frac{R}{r}\right)^{n+2} \sum_{m=o}^{n} [\ldots.]\, \overline{P}_{nm}(\cos\theta)$$

$$T_{rr} = \frac{\gamma}{R} \sum_{n=2}^{\infty} (n+1)(n+2)\left(\frac{R}{r}\right)^{n+3} \sum_{m=o}^{n} [\ldots.]\, \overline{P}_{nm}(\cos\theta)$$

T_{rr} est le gradient de gravité vertical (GOCE).