Beyond MHD:
Integration of Kinetic Effects in Multi-Fluid Global Magnetosphere Models

Amitava Bhattacharjee
Center for Heliophysics
Department of Astrophysical Sciences
and Princeton Plasma Physics Laboratory
Princeton University

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Center for Heliophysics, Princeton University/PPPL

- Amitava Bhattacharjee, *PI*, L. Comisso, Chuanfei Dong, Ammar Hakim, Yi-Min Huang, M. Lingam, Jonathan Ng, Jason TenBarge, Liang Wang

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Motivation

For global simulations of Earth's magnetosphere and space weather modeling, it is traditional to rely on **resistive** or at best, **Hall MHD**.

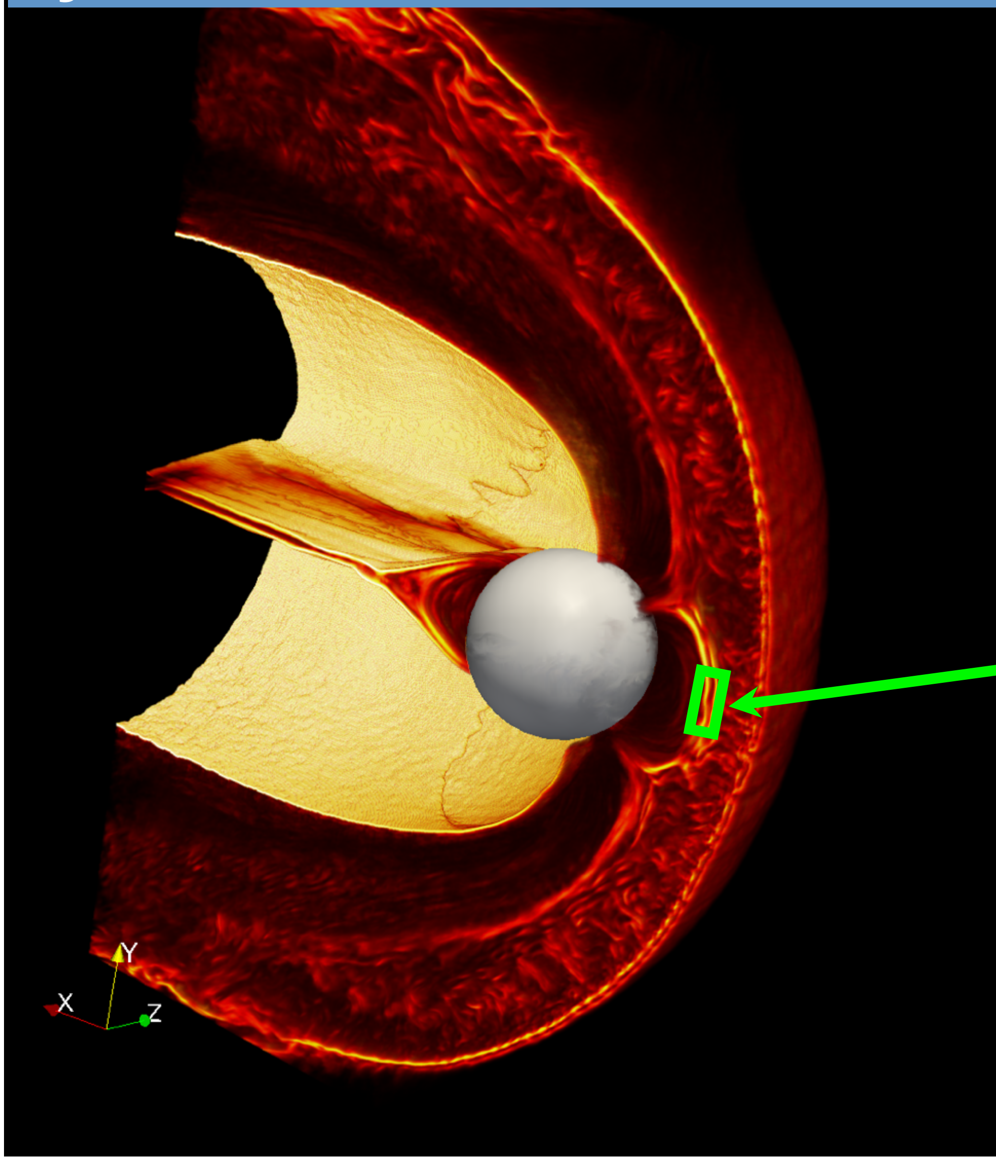
Common approach has been to use a generalized Ohm's law.

However, there seems to be no systematic way to add important collisionless kinetic effects in a self-consistent and numerically tractable manner for both electrons and ions.

A major challenge in the magnetosphere (and other applications) is that the **plasma is nearly collisionless** (large Knudsen number), and that the **magnetic fields (planetary dipole, solar wind) add a preferred direction**, adding significant global anisotropy to the system.

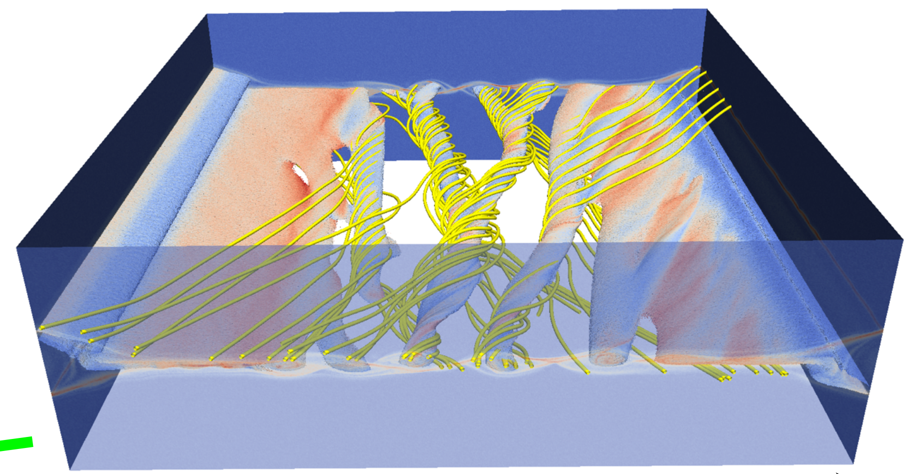
What simulations are feasible at the petascale?

Hybrid ~10^{10} cells ~ 10^{12} ions



Fully Kinetic

$\sim 10^{10}$cells $\quad \sim 10^{12}$ particles



$\sim 100 d_i \sim R_E$

$$3D \rightarrow m_i/m_e = 100 - 400$$

$$2D \rightarrow m_i/m_e = 400 - 1836$$

An alternative approach is to treat each plasma species with multi-fluid moment models with long-mean-free path closures

- In this approach we take moments of the Vlasov equation, truncating the moment sequence using a closure.
- The interaction between species is via electromagnetic fields, which are evolved using Maxwell equations (retaining displacement currents)
- This approach allows natural and self-consistent inclusion of **finite electron inertia, Hall currents, anisotropic pressure tensor and heat flux tensor**.
- Even though the multi-fluid moment equations contain physics all the way from light waves and electron dynamics to MHD scales, by use of advanced algorithms very efficient and robust schemes can be developed, **allowing us to treat a sequence of increasing fidelity models in a uniform and consistent manner**.

Foundations of *Gkeyll*: a new simulation approach

Multi-fluid models of plasmas are obtained by taking moments of Vlasov equation

Describe *each species* of the plasma as a finite set of moments of the Vlasov equation

$$\frac{\partial f}{\partial t} + v_j \frac{\partial f}{\partial x_j} + \frac{q}{m}(E_j + \epsilon_{kmj} v_k B_m) \frac{\partial f}{\partial v_j} = 0$$

Truncate the resulting moment system by a closure scheme. Evolve the electromagnetic fields with Maxwell equations, retaining displacement currents.

$$\frac{\partial \mathbf{B}}{\partial t} + \nabla \times \mathbf{E} = 0,$$

$$\mu_0 \epsilon_0 \frac{\partial \mathbf{E}}{\partial t} - \nabla \times \mathbf{B} = -\mu_0 \mathbf{J}$$

Sequence of models with 5, 10 and 20 moments

Taking moments of Vlasov equation leads to the *exact* moment equations listed below

$$\frac{\partial n}{\partial t} + \frac{\partial}{\partial x_j}(nu_j) = 0$$

$$m\frac{\partial}{\partial t}(nu_i) + \frac{\partial \mathcal{P}_{ij}}{\partial x_j} = nq(E_i + \epsilon_{ijk}u_jB_k)$$

$$\frac{\partial \mathcal{P}_{ij}}{\partial t} + \frac{\partial \mathcal{Q}_{ijk}}{\partial x_k} = nqu_{[i}E_{j]} + \frac{q}{m}\epsilon_{[ikl}\mathcal{P}_{kj]}B_l$$

$$\frac{\partial \mathcal{Q}_{ijk}}{\partial t} + \frac{\partial \mathcal{K}_{ijkl}}{\partial x_l} = \frac{q}{m}(E_{[i}\mathcal{P}_{jk]} + \epsilon_{[ilm}\mathcal{Q}_{ljk]}B_m)$$

In the **five-moment** model, we assume that the pressure is isotropic $P_{ij} = p\delta_{ij}$. For the **ten-moment** model, we include the time-dependent equations for all six components of the pressure tensor, and use a closure for the heat-flux. In the **twenty-moment** model, we evolve all ten components of the heat-flux tensor, closing at the fourth moment.

Five- and ten-moment models differ on how pressure and heat-flux are handled

Ten-moment model retains all six components of the pressure tensor. A self-consistent time-dependent equation is used

$$\partial_t P_{ij} + u_k \partial_k P_{ij} + P_{ij}\partial_k u_k + \partial_k u_{[i}P_{j]k} + \partial_k Q_{ijk} = \frac{q}{m} B_m \epsilon_{km[i}P_{jk]}$$

Square brackets around indices represent symmetrization. For example, $u_{[i}P_{j]k} = u_i P_{jk} + u_j P_{ik}$.

A closure is needed to determine $\partial_k Q_{ijk}$. One could use even higher moments[5], but some forms of higher moment equations have issues of realizability, i.e. may lead to distribution functions that are negative in some parameter space. Problem of closure does not go away.

Five moment model has $5S + 8$ equations, while ten-moment models have $10S + 8$ equations, where S is number of species.

Our presently implemented closure: local as well as non-local **Hammett-Perkins (1990)**

$$ik_m Q_{ijm}(k) = v_t |k| \tilde{T}_{ij}(k) n_0,$$

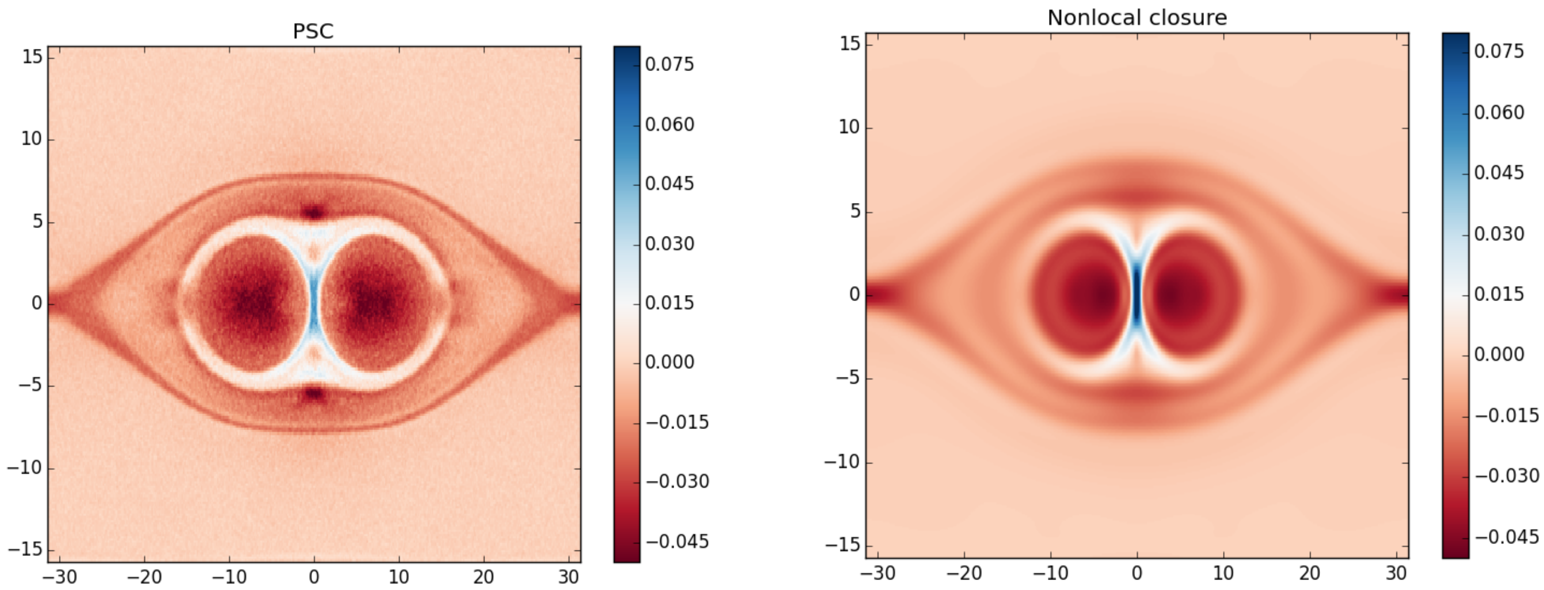
where, now, $k = |\mathbf{k}|$, $\tilde{T}_{ij}(k) = (\tilde{P}_{ij}(k) - T_0 \tilde{n} \delta_{ij})/n_o$

Local in k-space, but highly nonlocal in coordinate space

Nonlocal closure results – island coalescence challenge problem

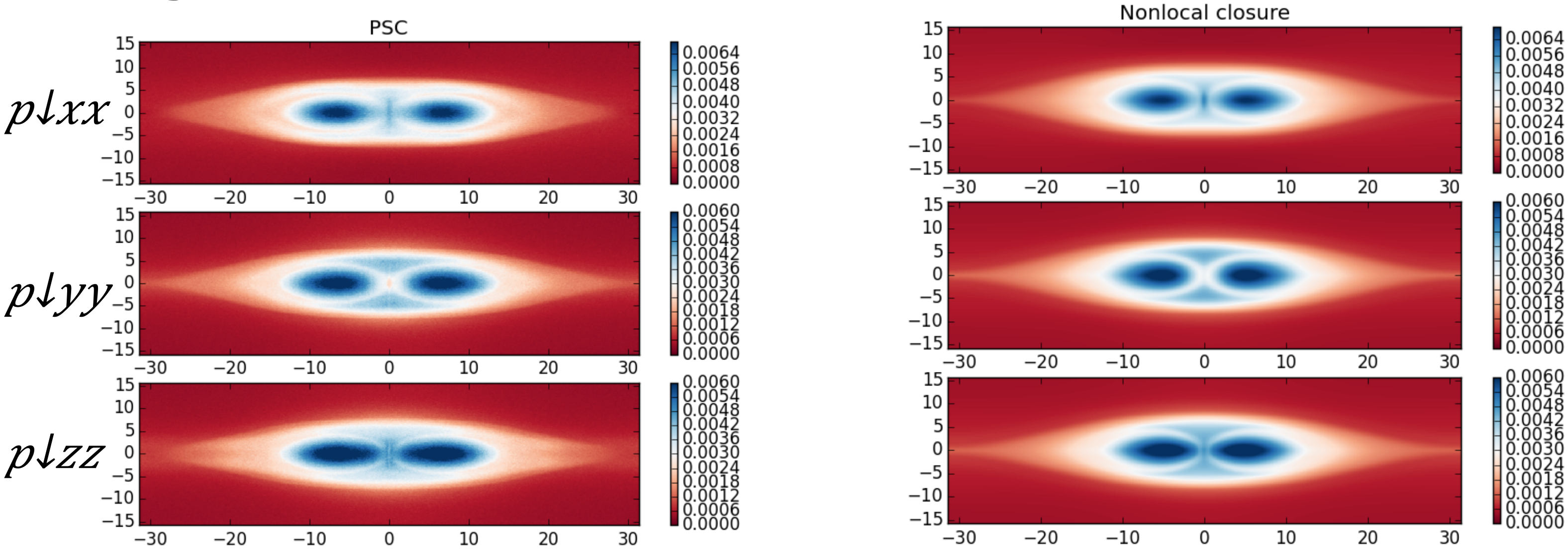
- 2-d model of "self-driven" reconnection
- Ion physics is important in setting the reconnection rate [Stanier 2015]

Current density at $t=t_A$

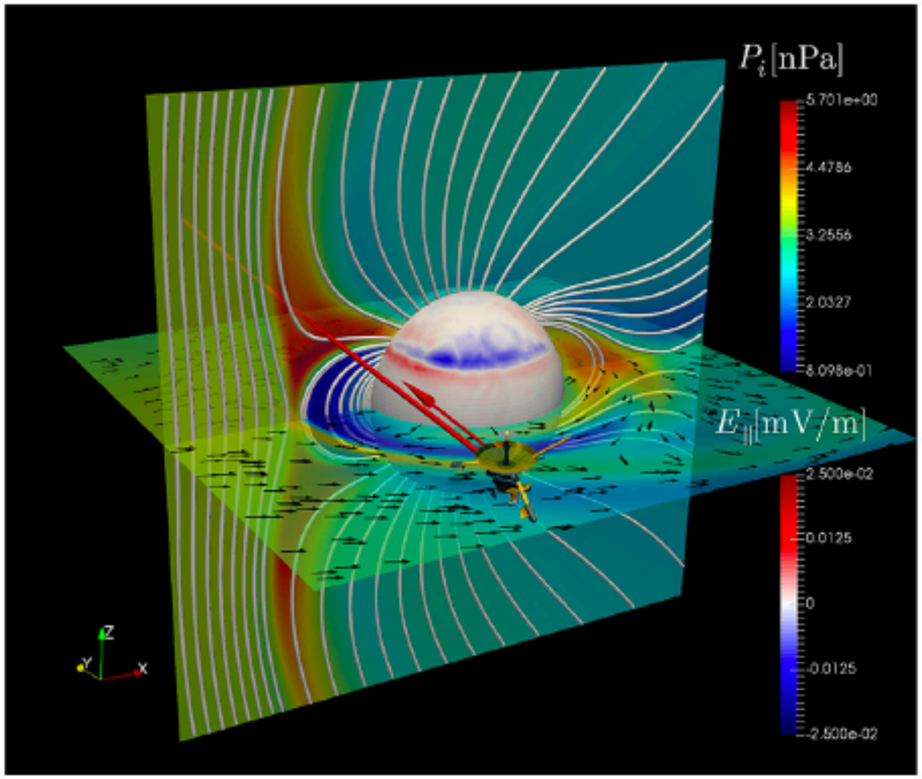


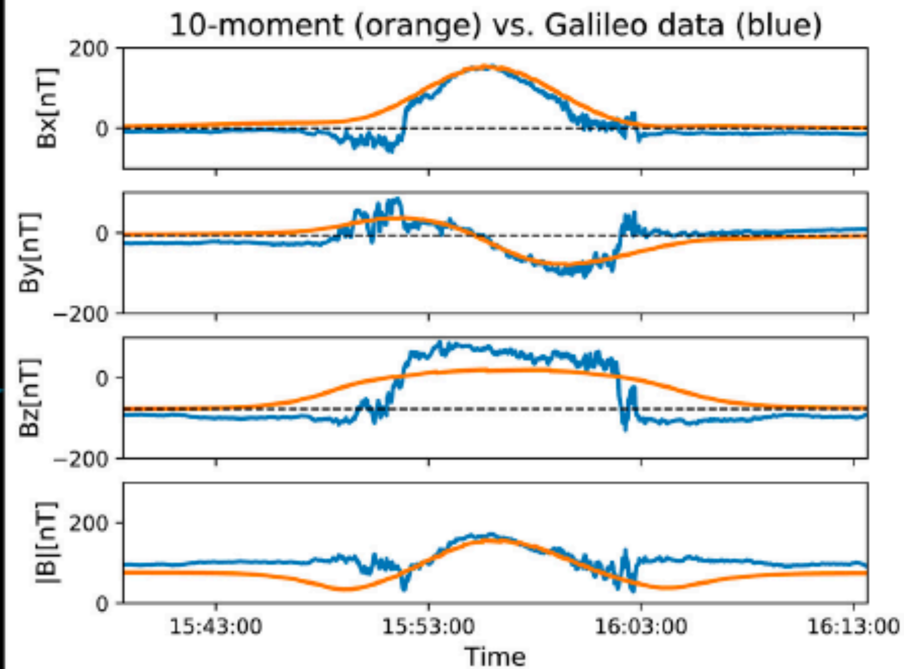
Diagonal components of ion pressure

- Heat flux at the x-point is crucial for better agreement with PIC



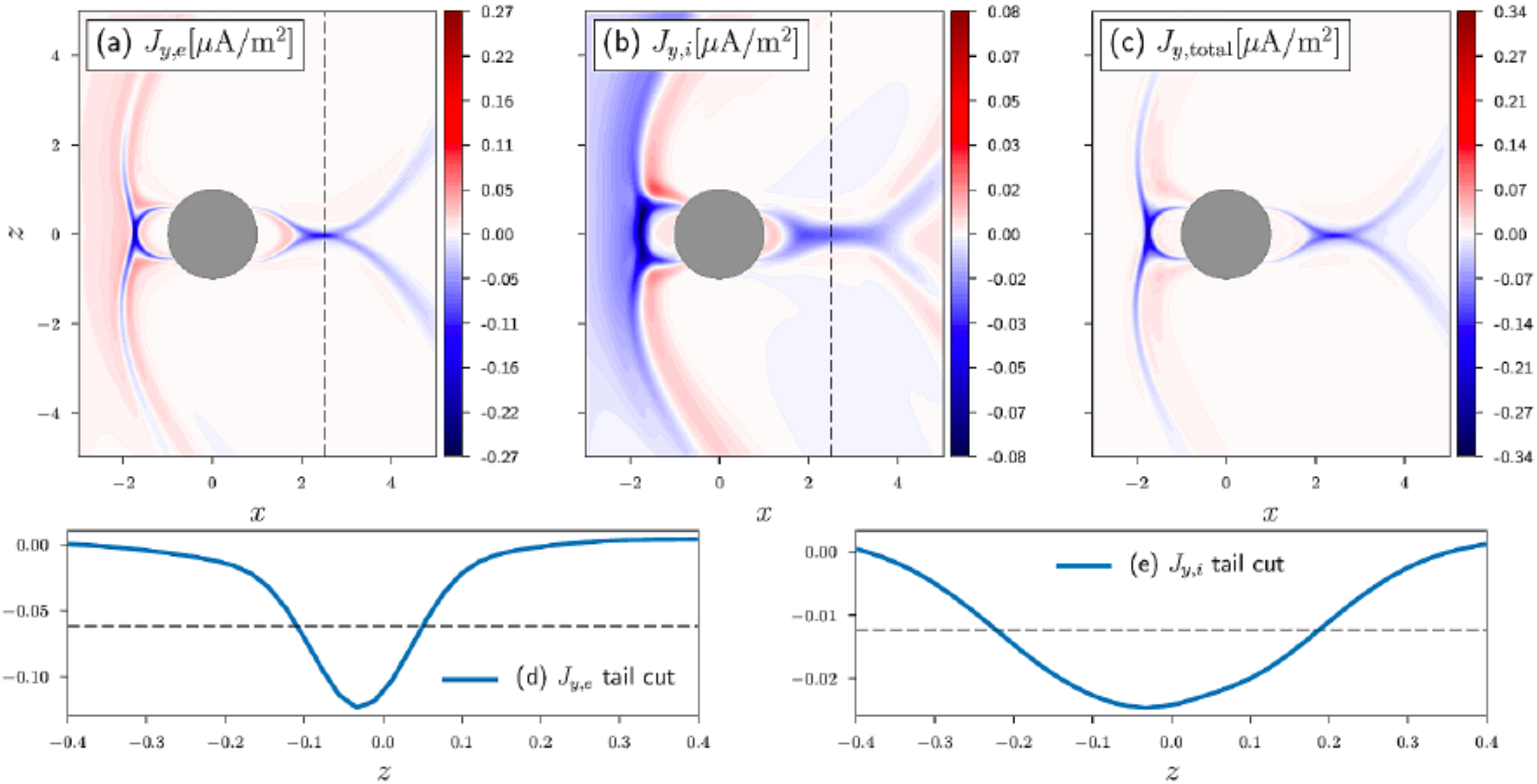
Validation: Magnetic field along Galileo G8 flyby trajectory





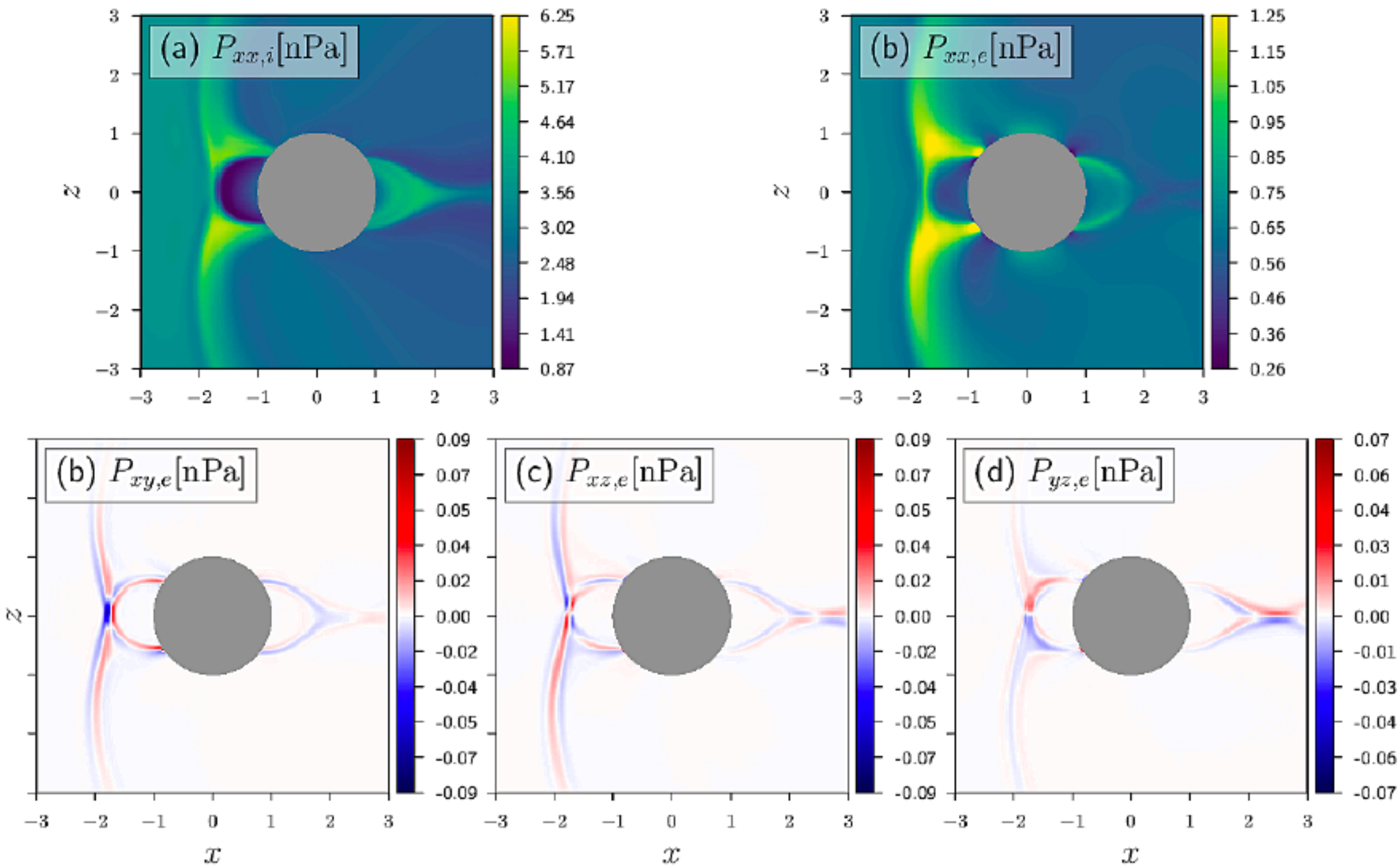
- B_x and B_y agree well with observation
- B_z deviates from observation, possibly caused by
  - overly simplified inner boundary condition; more realistic conductance layer might be required (e.g., [Jia et al., 2009; Sour et al., 2016])
  - insufficient resolution near the magnetosphere crossings

Reconnection physics 1: Current sheet formation



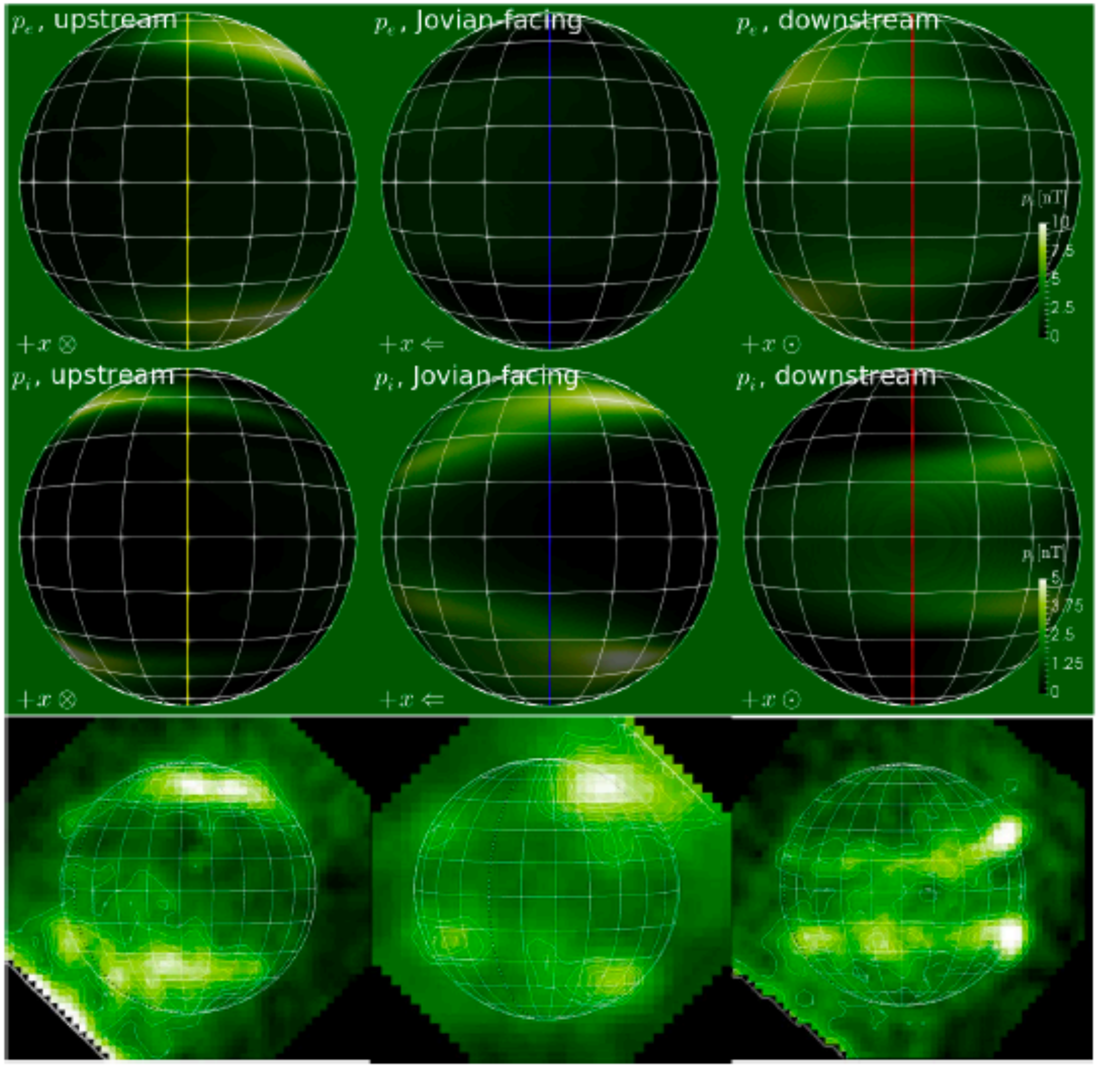
- dense currents carried mainly by electrons
- $J_{y,e}$ half-thickness $\sim 2d_{e0}$, $J_{y,i}$ half-thickness $\sim 1d_{i0}$

Reconnection physics 2: FLR-effect and Pressure tensors



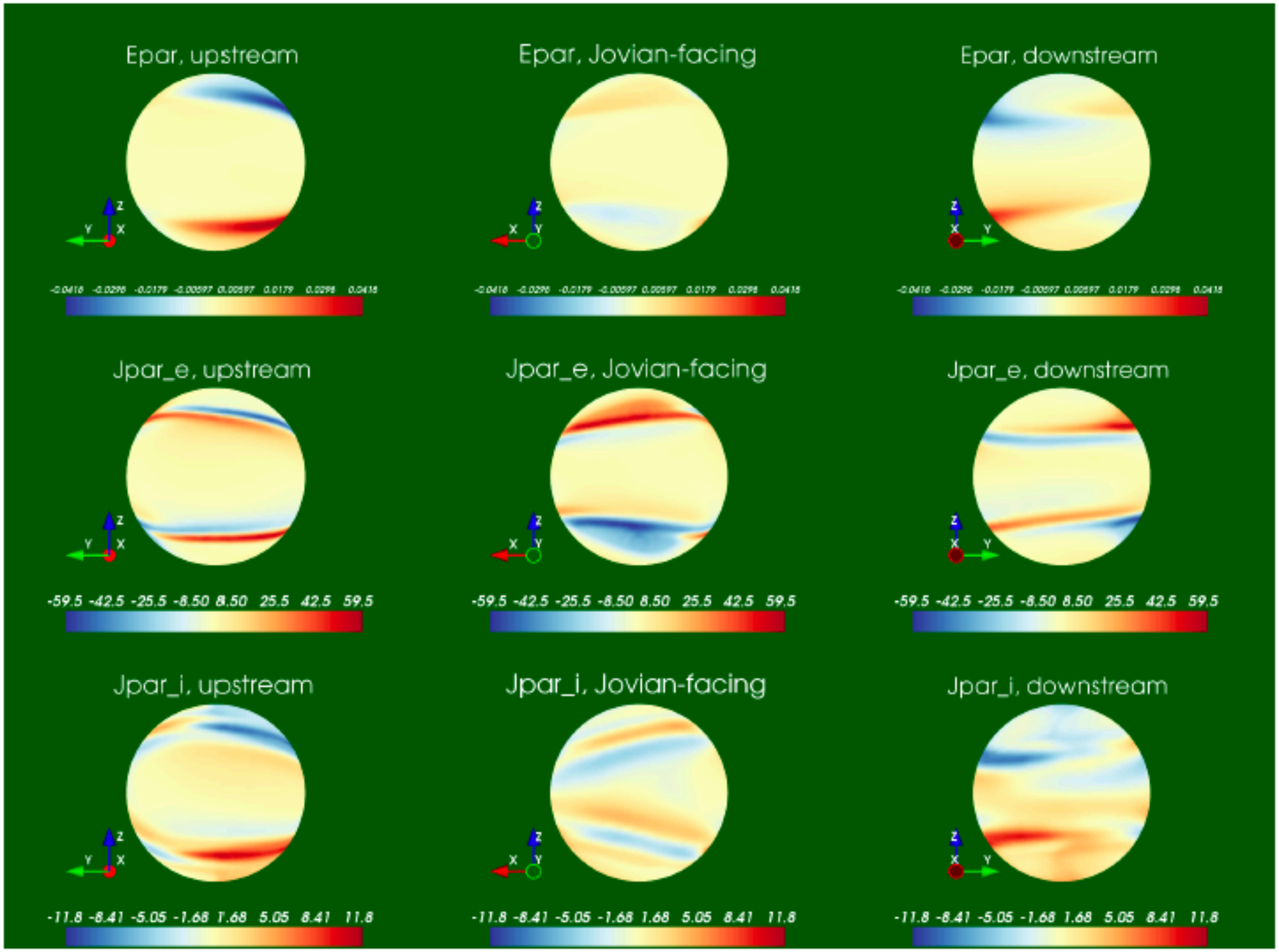
- $\mathbf{P}_{off,e}$ polarities agree with local 2D PIC/Vlasov studies

Surface morphologies 1: pressure vs. HST brightness observation



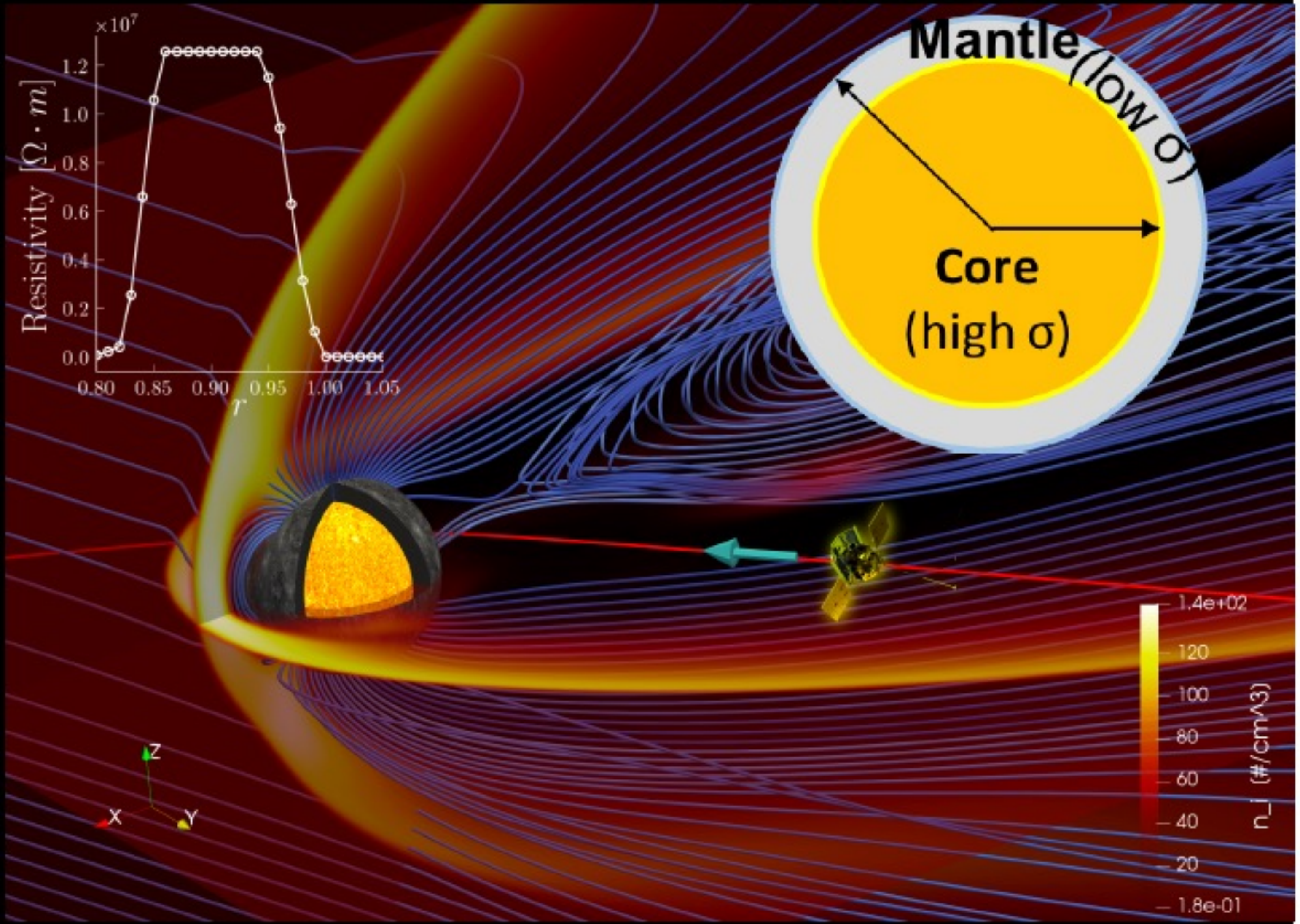
- First and second row: Surface electron and ion pressure contours
- Third row: Oxygen emission observed by Hubble Space Telescope
- The three columns are for the upstream, Jovian-facing, and downstream hemispheres

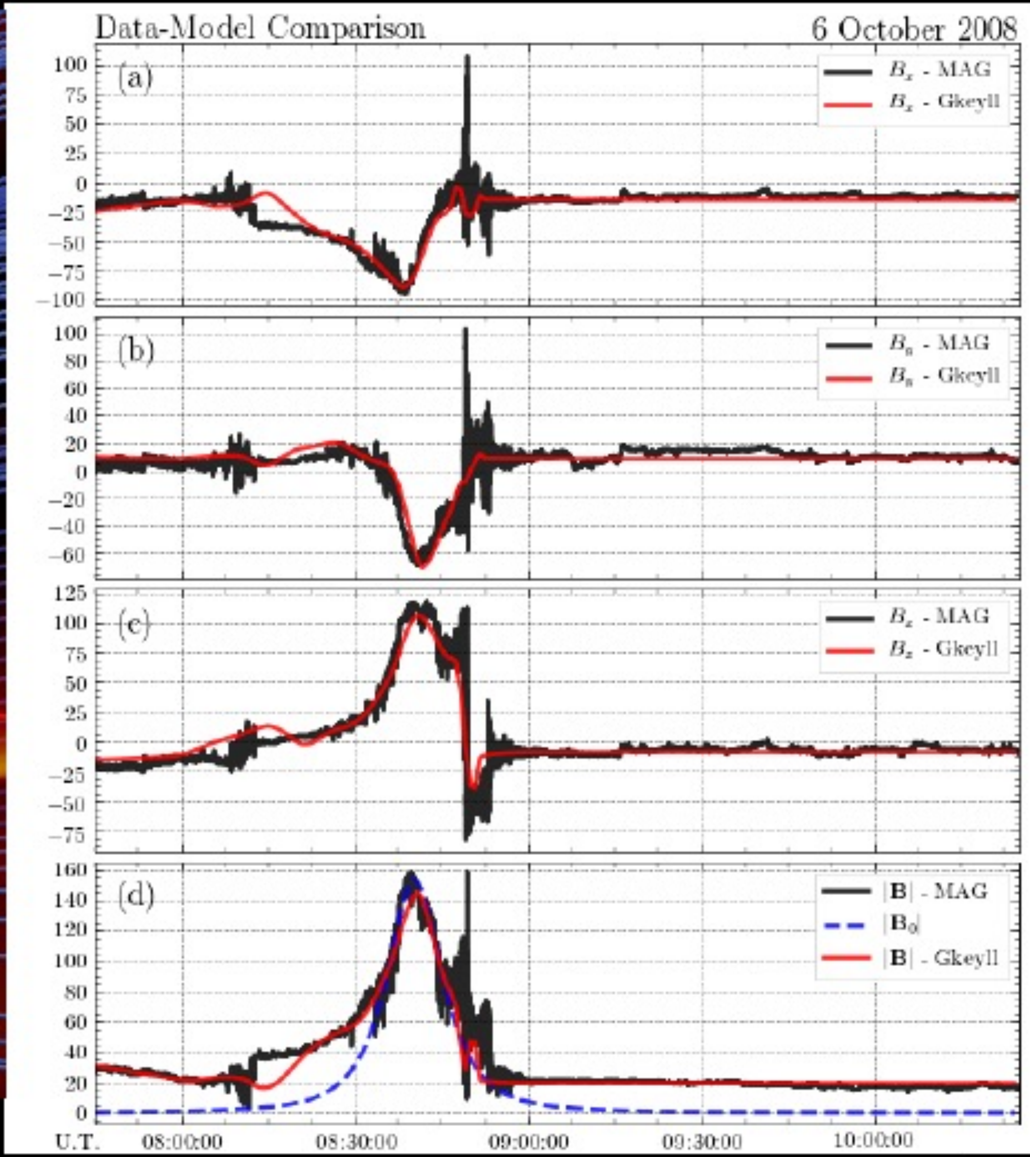
Surface morphologies 2: Surface $J_\|$ and $E_\|$



- First and second row: Surface electron and ion current density
- Third row: Surface parallel electric field

Ten-Moment Multifluid Simulation of Mercury's Magnetosphere





Smallest resolution: $dx=dy=dz=0.01R_M$!

Dong et al., 2019, GRL

An MMS Event Study of 3D Asymmetric Reconnection with Extended MHD

J. M. TenBarge
Amitava Bhattacharjee, Ammar Hakim, Jimmy Juno, Jonathan Ng, Liang Wang

In this study, we employ the Gkeyll simulation framework to study the Burch event with different classes of extended magnetohydrodynamics (MHD), including multi-fluid models that incorporate important kinetic effects such as electron pressure tensor (with physics-based closure relations) and Landau damping.