Rice Convection Model:
An update

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RCM

RCM solves the guiding-center Vlasov equation for isotropic distribution function of hot plasma in the inner magnetosphere:

$$\left(\frac{\partial}{\partial t}+\vec{V}_D\cdot\nabla\right)f(\lambda,\vec{x},t)=S-L,\quad \text{where}\quad \vec{V}_D=\frac{\vec{B}\times\vec{\nabla}\left(q\Phi'+\lambda V^{-2/3}\right)}{qB^2}$$

subject to constraint:

$$\nabla\cdot\left(-\Sigma_p\nabla\Phi\right)=J_{\|}$$

where the two equations are connected via P (moment of *f*):

$$\frac{J_{\|in}-J_{\|is}}{B_i}=\frac{\hat{\mathbf{b}}\cdot\vec{\nabla}V\times\vec{\nabla}P}{B}$$

Magnetic field and PSD *f* are either prescribed with empirical models (standalone RCM) or provided by global MHD codes (coupled MHD-RCM: **SWMF with RCM**, OpenGGCM, LFM-RCM).

Modeler's Paranoia: Code Verification

- Does the code solve the equations it claims to solve?
  - Analytic solutions
- Do the numerical schemes in the code work correctly? Under what conditions (spatial grid and time resolution) do they break down?
  - Analytic solutions
  - Diagnostics:
    - energy conservation
    - entropy conservation
    - pressure balance in quasi-steady state, etc
- Does code work correctly in other aspects?
  - Interpolation schemes
  - Code modifications (reference solutions)
- Alternative approach: Do solutions resemble observations?

RCM Code Verification

- We designed a series of analytic test cases that test each of the three modules separately (work in progress):
  - Advection equation (2 tests):

$$\left(\frac{\partial}{\partial t}+\vec{V}_D\cdot\nabla\right)f(\lambda,\vec{x},t)=S-L,\quad \text{where}\quad \vec{V}_D=\frac{\vec{B}\times\vec{\nabla}\left(q\Phi'+\lambda V^{-2/3}\right)}{qB^2}$$

  - Potential solver (3 tests)

$$\nabla\cdot\left(-\Sigma_p\nabla\Phi\right)=J_{\parallel}$$

  - Field-aligned currents calculation (1 test)

$$\frac{J_{\parallel in}-J_{\parallel is}}{B_i}=\frac{\hat{\mathbf{b}}\cdot\vec{\nabla}V\times\vec{\nabla}P}{B}$$

- Code can be configured with only one of the three modules installed, and the other two are replaced by “test” modules specifying analytic test cases.

Test case for potential solver

Look for a solution to the equation $\nabla \cdot \left(-\Sigma_p \nabla \Phi \right) = J_{\parallel}$
on the sphere with no Hall conductance where there is a fairly sharp jump in Σ_p at the terminators ($\phi=\pi/2$ and $3\pi/2$). No latitudinal dependence.

Choose:

$$\frac{1}{\Sigma_p} = \frac{1}{\Sigma_d} + \frac{1}{2}\left(\frac{1}{\Sigma_n} - \frac{1}{\Sigma_d} \right)\left[\tanh\left(K(\phi - \frac{\pi}{2}) \right) - \tanh\left(K(\phi - \frac{3\pi}{2}) \right) \right]$$

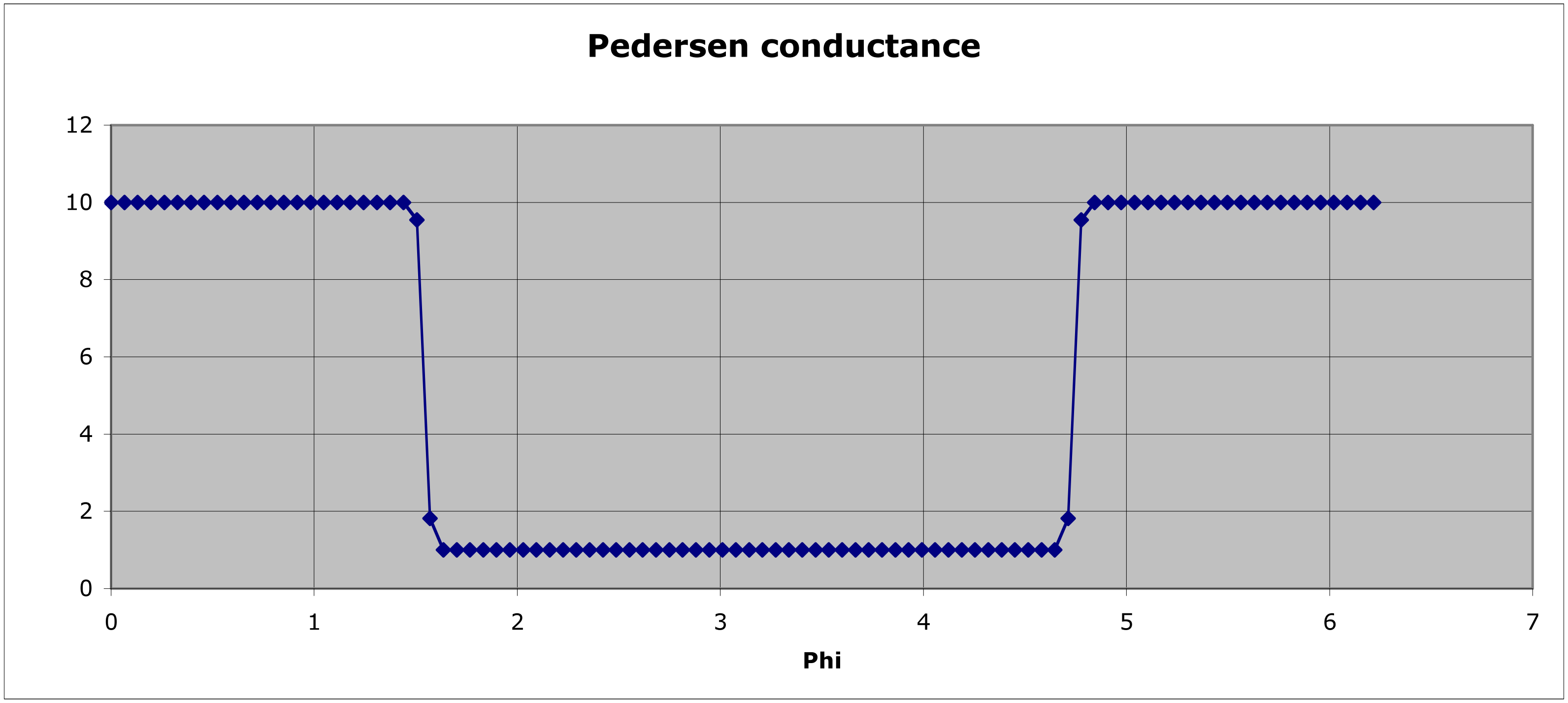
with Σ_d=10, Σ_n=1, K=40 (96 azimuthal grid points in RCM),

$$J_{\parallel}(\theta,\phi) = \frac{J_o K'}{\sin^2\theta}\left\{ \mathrm{sech}^2\left[K'\phi \right] - \mathrm{sech}^2\left[K'(\phi - \pi) \right] \right\}$$

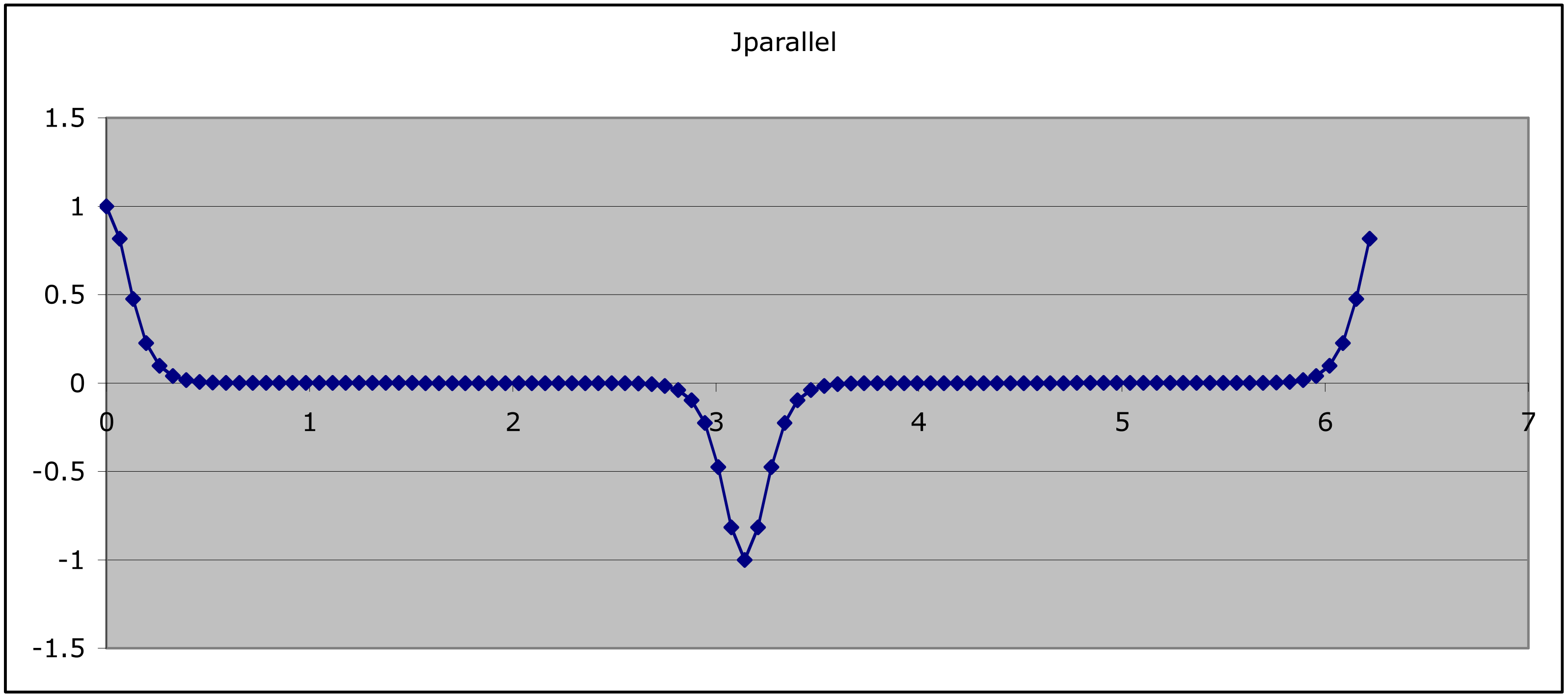
with J_o = 1, K'=7, and

Specify polar boundary potential $\Phi_{pb}(\phi)$

Test case for potential solver

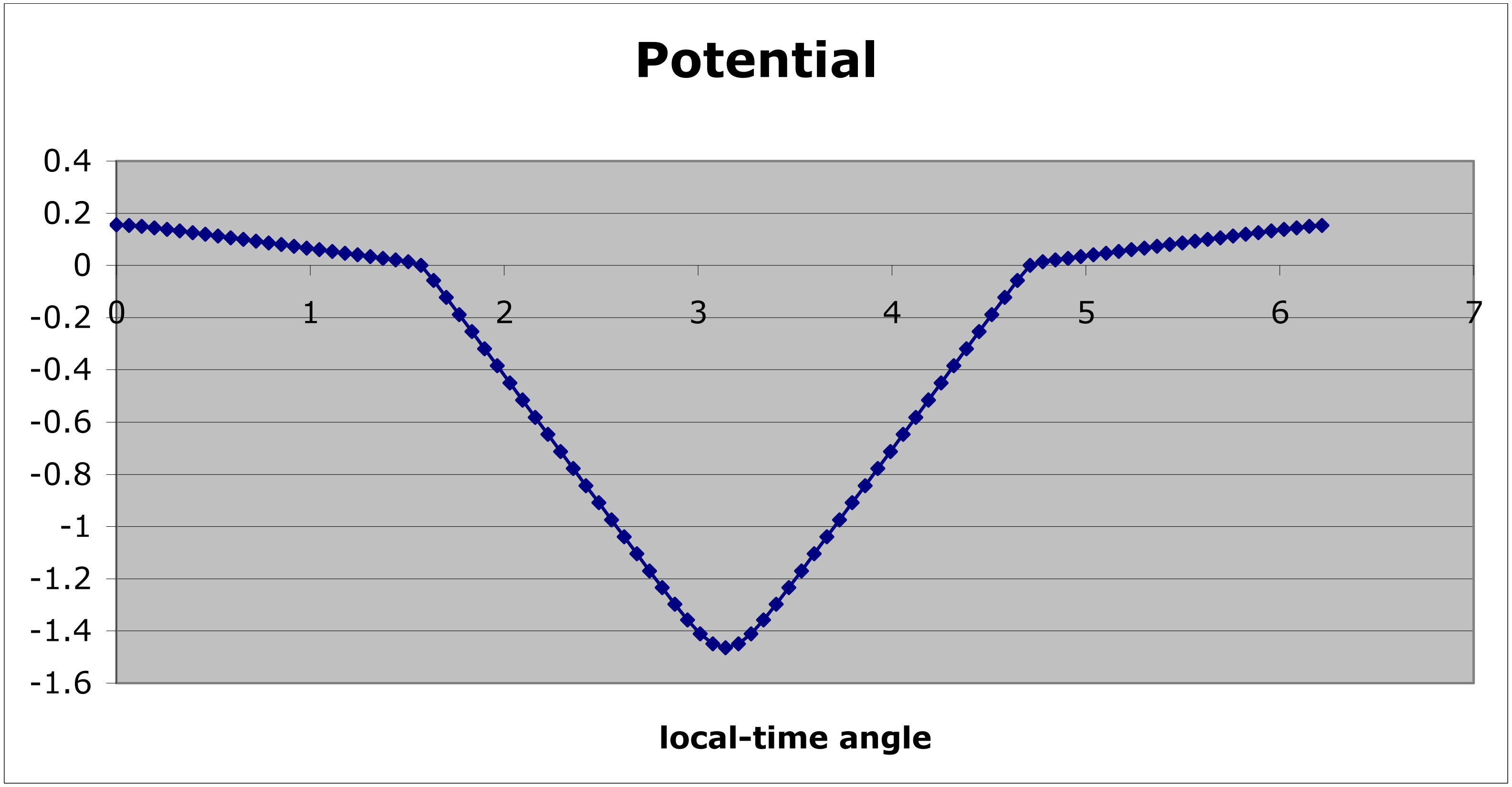


Test case for potential solver

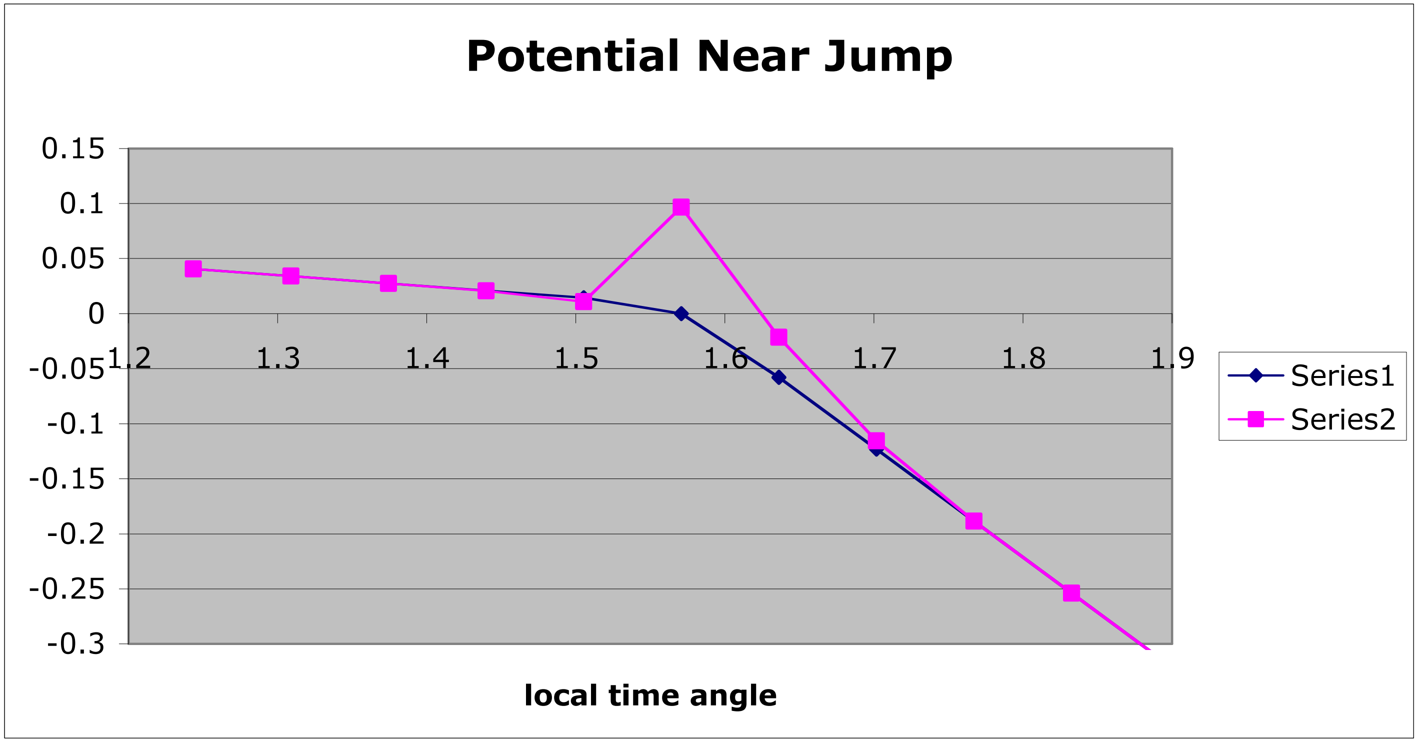


Test case for potential solver

$$\Phi(\theta,\phi) = \Phi_{pb}(\phi)$$



Analytic vs Numerical Solution: $\Phi(\theta,\phi) = \Phi_{pb}(\phi)$



Blue curve (analytic solution). Pink curve (numerical solution). Potential very close to the terminator is clearly wrong, with an unphysical reversed electric field between two grid points.

Field-Line Integrals in Studying Plasma Sheet Transport

- In the RCM formalism (drift physics), there is conservation law:

  $V = \int \frac{ds}{B}$ is flux tube volume, P is pressure

  $$PV^{\gamma} = \text{const}$$

  leading to “pressure balance inconsistency”.

- In nature, non-adiabatic processes violate the entropy parameter PV^{γ}. There is lots of observational evidence associating bursty bulk flows with bubbles (e.g., *Sergeev et al*., *JGR,* 1996; *Nakamura et al*., *JGR,* 2001).

- In global MHD-RCM codes, violation of PV^{γ} has to occur outside the inner magnetosphere (in the tail plasma sheet). Obvious causes are (numerical) reconnection, numerical resistivity/instability (e.g., Raeder et al. [2010]), numerical errors.

Field-Line Integrals in Studying Plasma Sheet Transport

$$V = \int \frac{ds}{B} \text{ is flux tube volume}$$

$$M = \int \rho \frac{ds}{B} = \text{mass per unit magnetic flux}$$

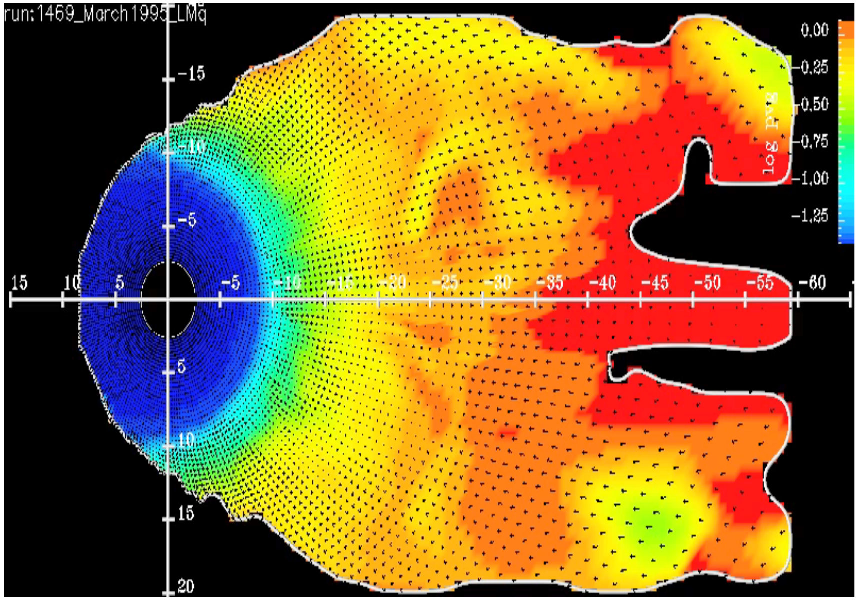
$$S = \int P^{1/\gamma} \frac{ds}{B} = \int \left(\frac{P^{1/\gamma}}{\rho} \right) \frac{\rho\, ds}{B} = \int e^{(\gamma-1)\sigma/\gamma} \left(\frac{\rho ds}{B} \right) = \text{entropy parameter}$$

σ=entropy per particle, and $\rho\ ds/B$ =mass element.

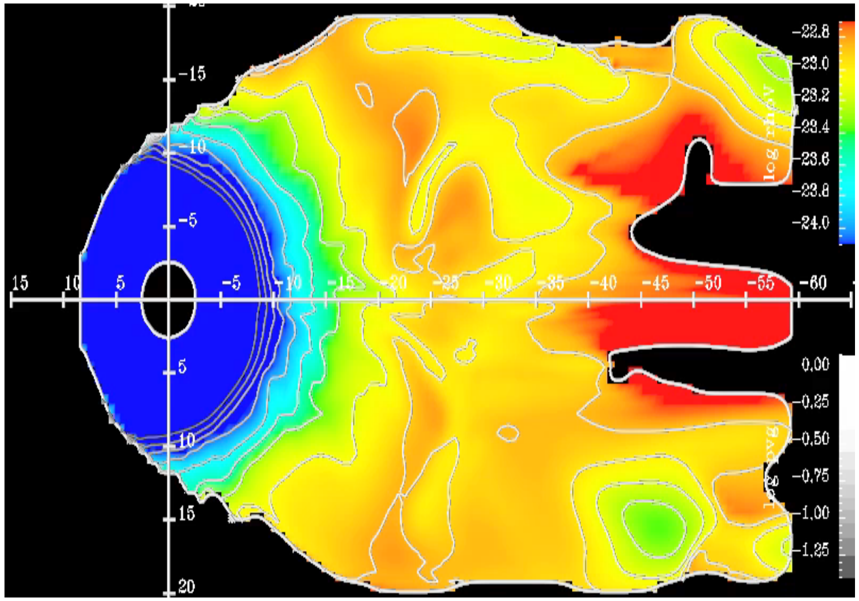
- Both M and S are conserved in ideal MHD as a flux tube drifts.
- V and S are crucial for simple interchange instability, for which the criterion is $\delta S\ \delta V<0$.

Movies of equatorial distributions of S seem to illuminate the physics of plasma sheet transport, as illustrated in the following two examples:

Plasma Sheet Transport in LFM



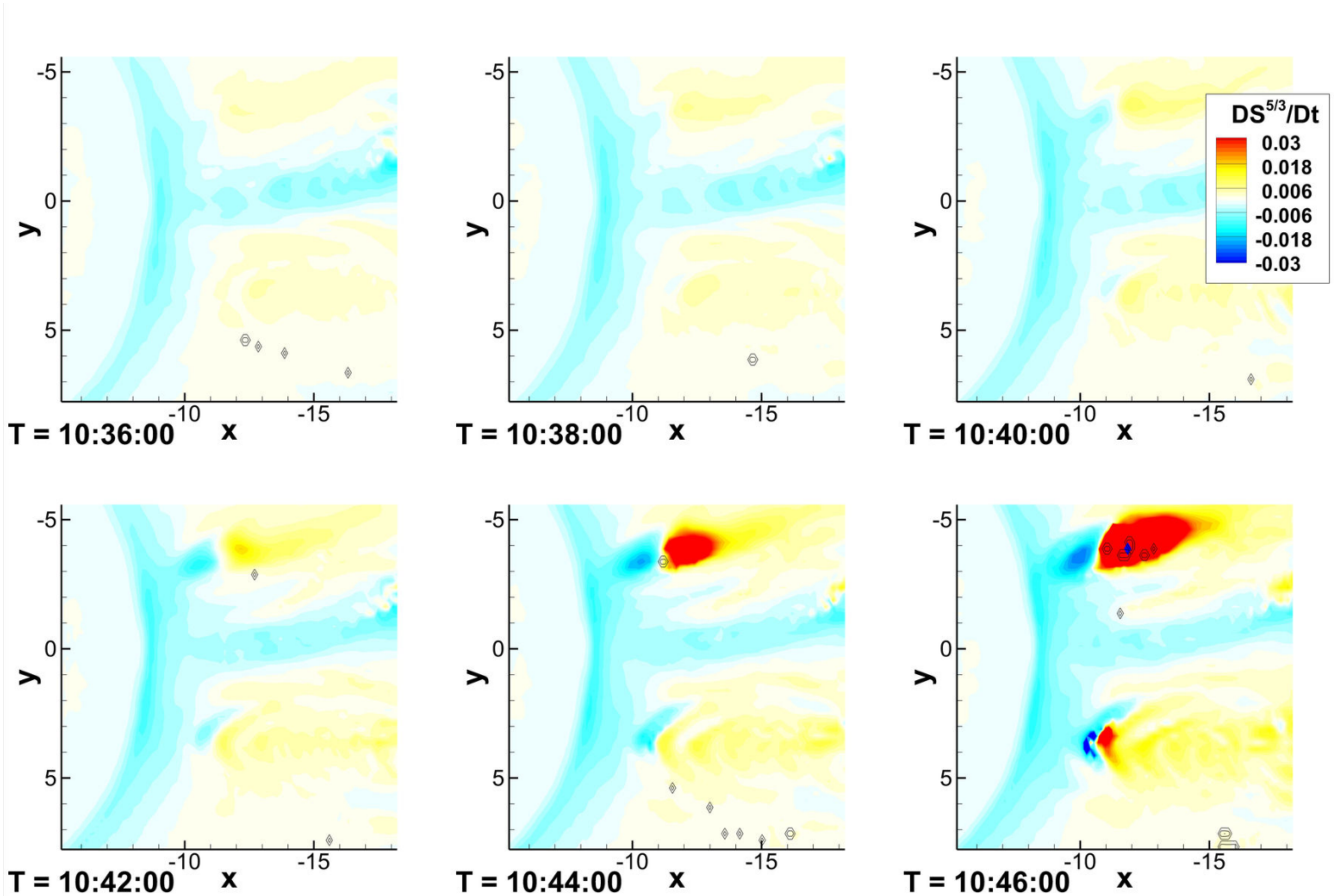
$S^{5/3}$ and velocity



M

- LFM-based movie courtesy of Asher Pembroke; also *Pembroke et al. JGR* accepted
- In the left plot, green and blue regions (low $S^{5/3}$, bubbles) move systematically sunward through the plasma sheet.
- Regions of low *S* (bubbles) also tend to be regions of low mass *M*.
- Plasma sheet transport is dominated by fast-flowing bubbles.

Reduction in $S^{5/3}$ precedes reconnection in OpenGGCM



Numerical error consistently reduces $S^{5/3}$ near plasma sheet inner edge and in flow channel near midnight. Creation of a bubble-blob pair led to reconnection and substorm onset in the model. OpenGGCM simulation from *Hu et al*. [*JGR*, 2011].)

Suggestion

If CCMC offered the capabilities for computing field line integrals and displaying the results in the equatorial plane, for example, that would:

- Facilitate use of global-MHD codes for studying plasma sheet transport
- Provide information on how accurately the global MHD codes solve their partial differential equations.

Summary

- Development of a test suite of analytic solutions for RCM to test if the code actually solves the RCM equations.
- In coupled global MHD-RCM codes (and standalone MHD), quantities key to transport physics are field-line integrals.
- It would be wonderful if CCMC could provide the capability for computing field-line integrals
  - Based on results from global-MHD runs.
  - Display in equatorial plane (and maybe ionosphere).

$$V = \int \frac{ds}{B} = \text{flux tube volume}$$

$$M = \int \rho \frac{ds}{B} = \text{mass per unit magnetic flux}$$

$$S = \int P^{1/\gamma} \frac{ds}{B} = \int \left(\frac{P^{1/\gamma}}{\rho} \right) \frac{\rho\, ds}{B} = \int e^{(\gamma-1)\sigma/\gamma} \left(\frac{\rho ds}{B} \right) = \text{entropy parameter}$$