Empirical Ionospheric Electrodynamic Models at the CCMC

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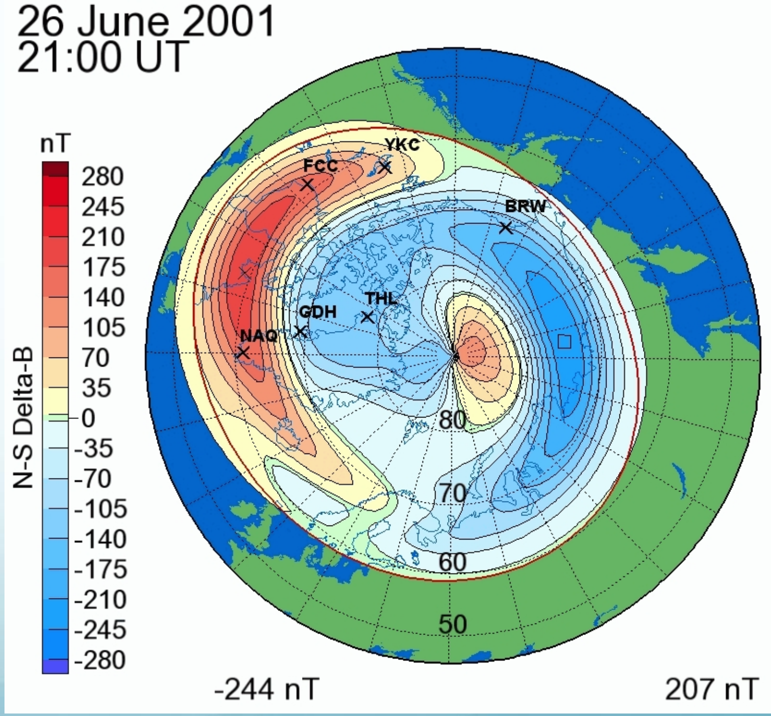
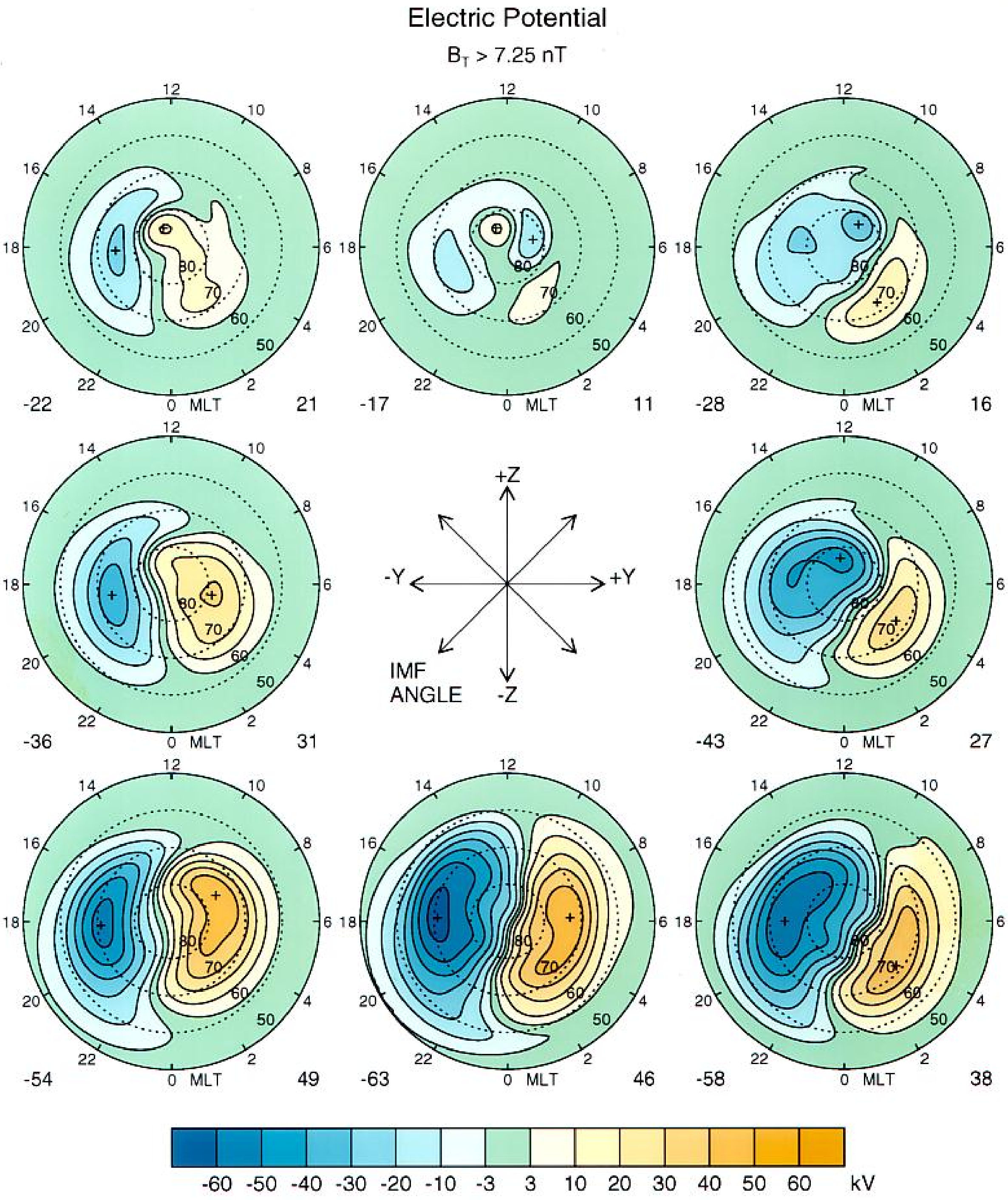


Using electric field/potential measurements from the DE-2 satellite, a 1995 publication used a least-error fit of spherical harmonic coefficients to derive the potential patterns from the sparse and randomly distributed measurements.

The passes were sorted into "bins" by IMF magnitude, clock angle, and dipole tilt angle.

A fixed, low-latitude boundary of 45° was used.



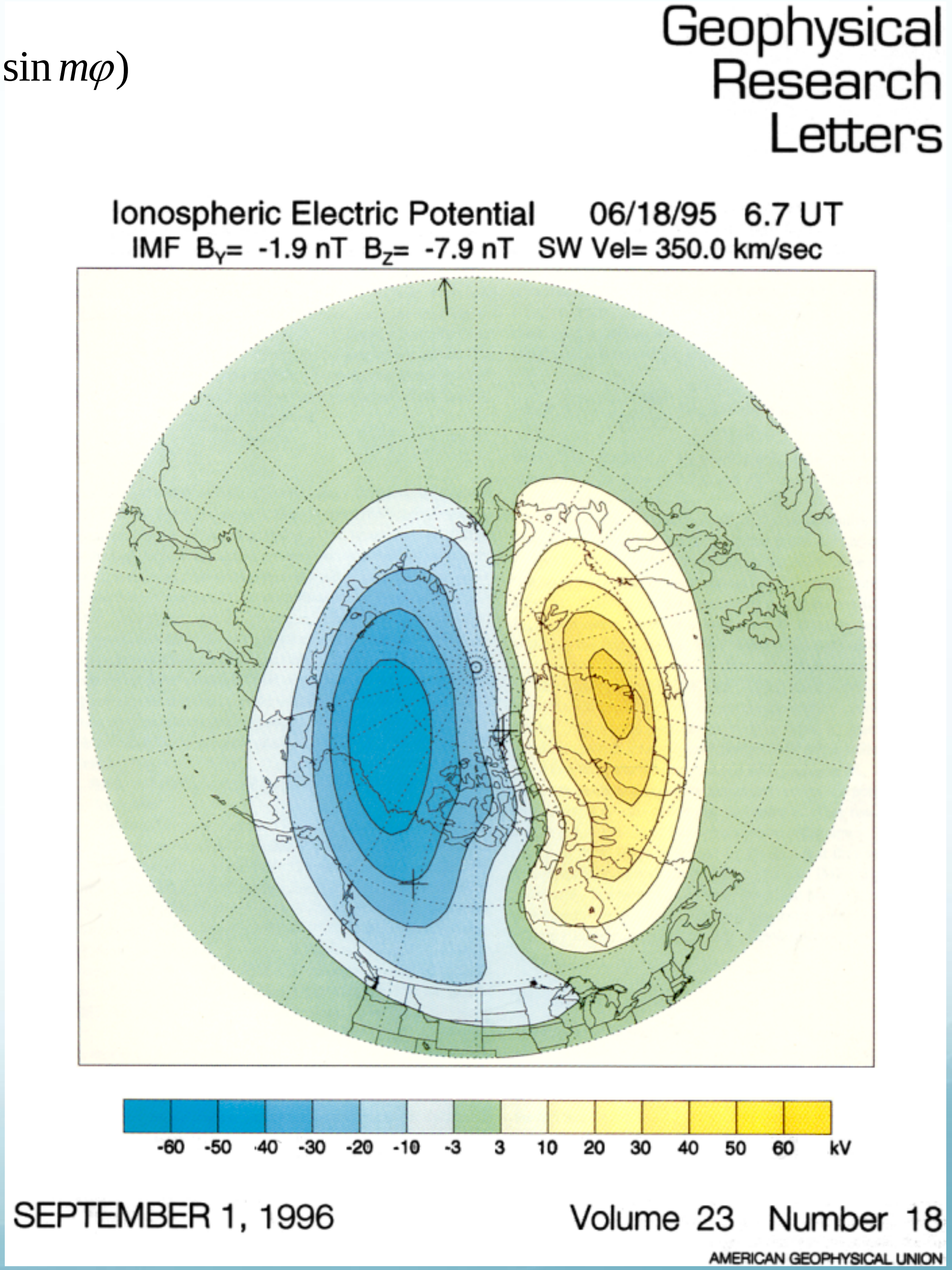
$$\Phi(\Lambda,\varphi) = \sum_{l=0}^{8} \sum_{m=0}^{Min(3,l)} P_l^m(\cos\Lambda)(A_l^m \cos m\varphi + B_l^m \sin m\varphi)$$

$$B_{11}(\omega) = \sum_{n=0}^{4}(C_n \cos n\omega + D_n \sin n\omega)$$

$$C_{B_{lm}n} = R_0 + R_1 B_T + R_2 \sin\mu + R_3 V_{SW}$$

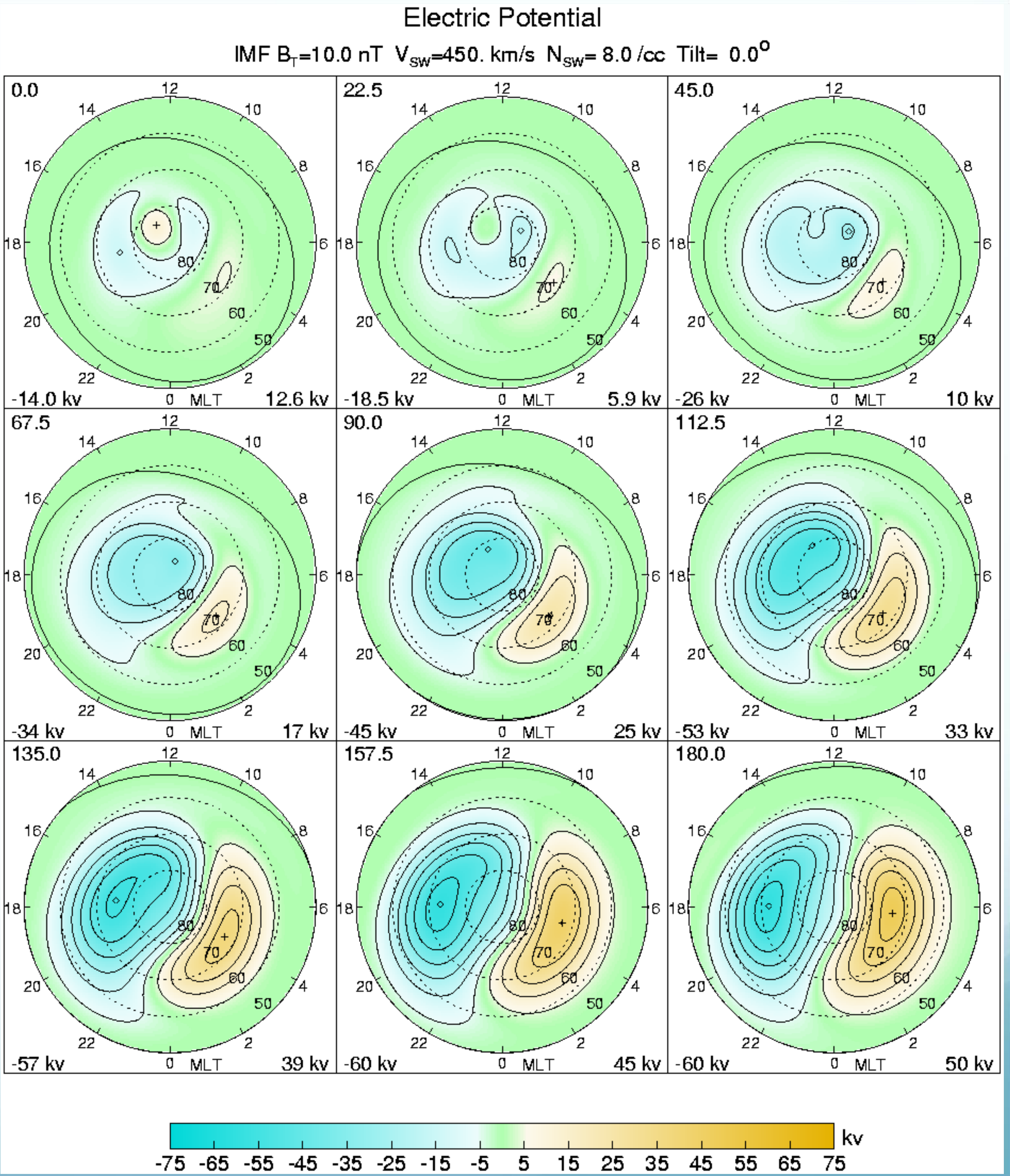
The 1996 model could create a potential map for any arbitrary IMF.

The spherical harmonic coefficients from the sorted, “binned” patterns, with their average IMF values, were used as the inputs to the model construction.



The 2001 version of the model had several improvements:

- Added a non-linear response, proportional to $B_T^{2/3}$.
- Used an expanding and contracting lower boundary, rather than fixed.
- Added an optional term controlled by the AL Index.



Another 2001 publication describes a field-aligned current (FAC) model that avoids the "infinite current sheet approximation." With a radial current, the magnetic field on an orthogonal surface is such that *:

$$\Delta \mathbf{B} = \hat{\mathbf{r}} \times \nabla_S \psi$$

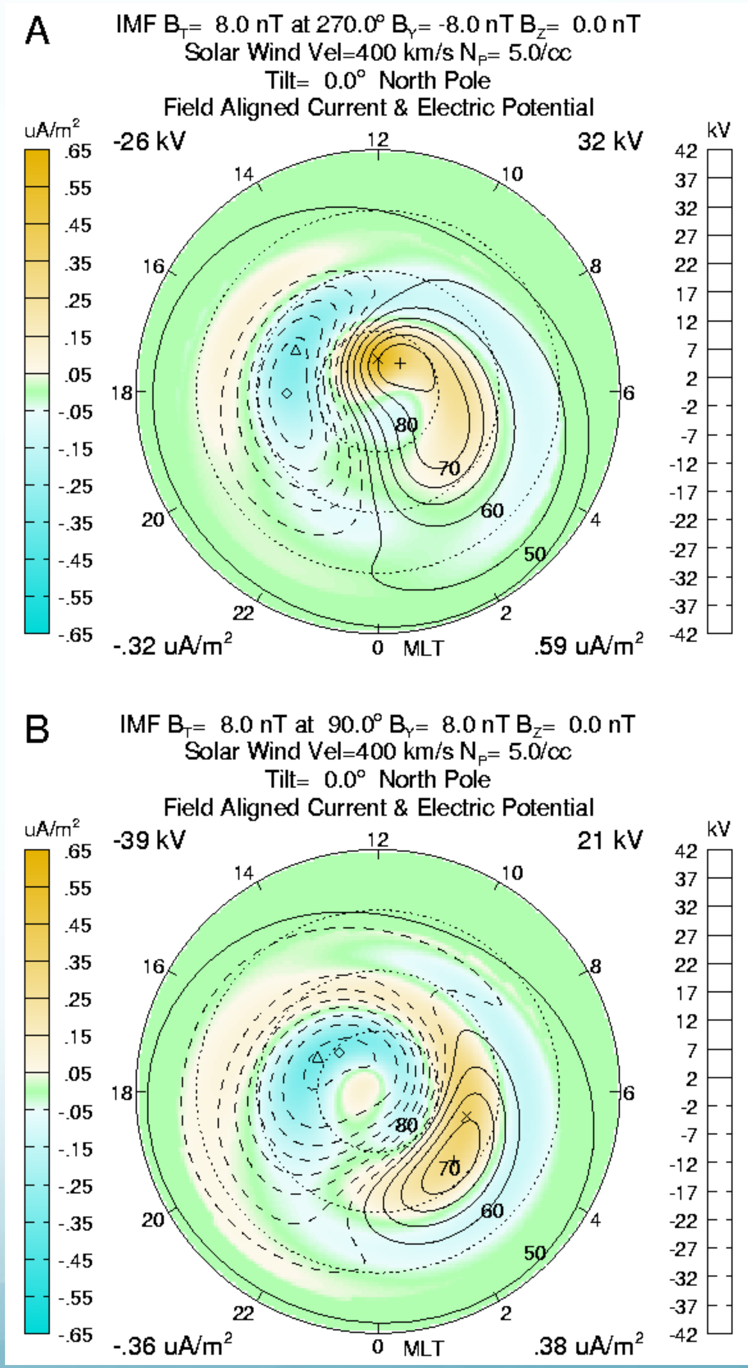
$$\mu_o \mathbf{J} = \nabla \times \Delta \mathbf{B} = \nabla \times (\hat{\mathbf{r}} \times \nabla_S \psi) = \hat{\mathbf{r}} \nabla_S^2 \psi$$

$$J_{\parallel} = \nabla_S^2 \psi / \mu_o$$

This FAC model, based on "magnetic Euler potentials" from DE-2 magnetometer data, is coded very much like the electric potential model.

The figures on the right show the FAC for + and – IMF B_Y clock angles, superimposed with the electric potential contours.

*Backus, G., Poloidal and toroidal fields in geomagnetic field modeling, *Rev. Geophys.*, *24*, 75, 1986. (Incidentally, the APL-NSF "AMPERE" project uses the same methodology.)



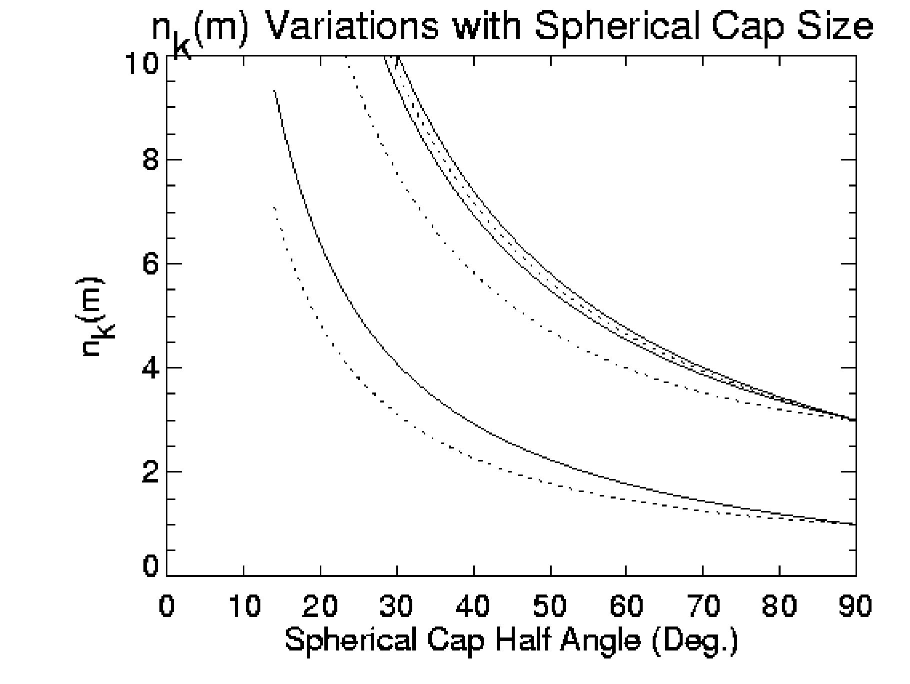
A 2005 revision to both models improved performance and added features:

- Both models now share same program code and have a common, expandable low-latitude boundary. Spherical cap offset 4.2^{o} from pole.
- New IMF propagation model was used to revise data base.
- Adds Joule heating, and calculation of ground-level magnetic perturbations.

Using higher-degree "spherical cap harmonic analysis" * (SCHA);
Associated Legendre functions have real, non-integer degree, $n_k(m)$, that depends on the integer order m and degree k, as well as the polar cap half angle, θ_o.

Analytic formulas may be used for derivatives.

*Haines, G. V., Spherical cap harmonic analysis, *J. Geophys. Res., 90, B3*, 2583, 1985.



A non-linear "saturation" response is obtained by an exponential function of the interplanetary electric field:

$$\psi(\Lambda,\varphi) = \sum_{k=0}^{12} \sum_{m=0}^{2<k} P_{nk(m)}^{m}(\cos\Lambda)(g_k^m \cos m\varphi + h_k^m \sin m\varphi)$$

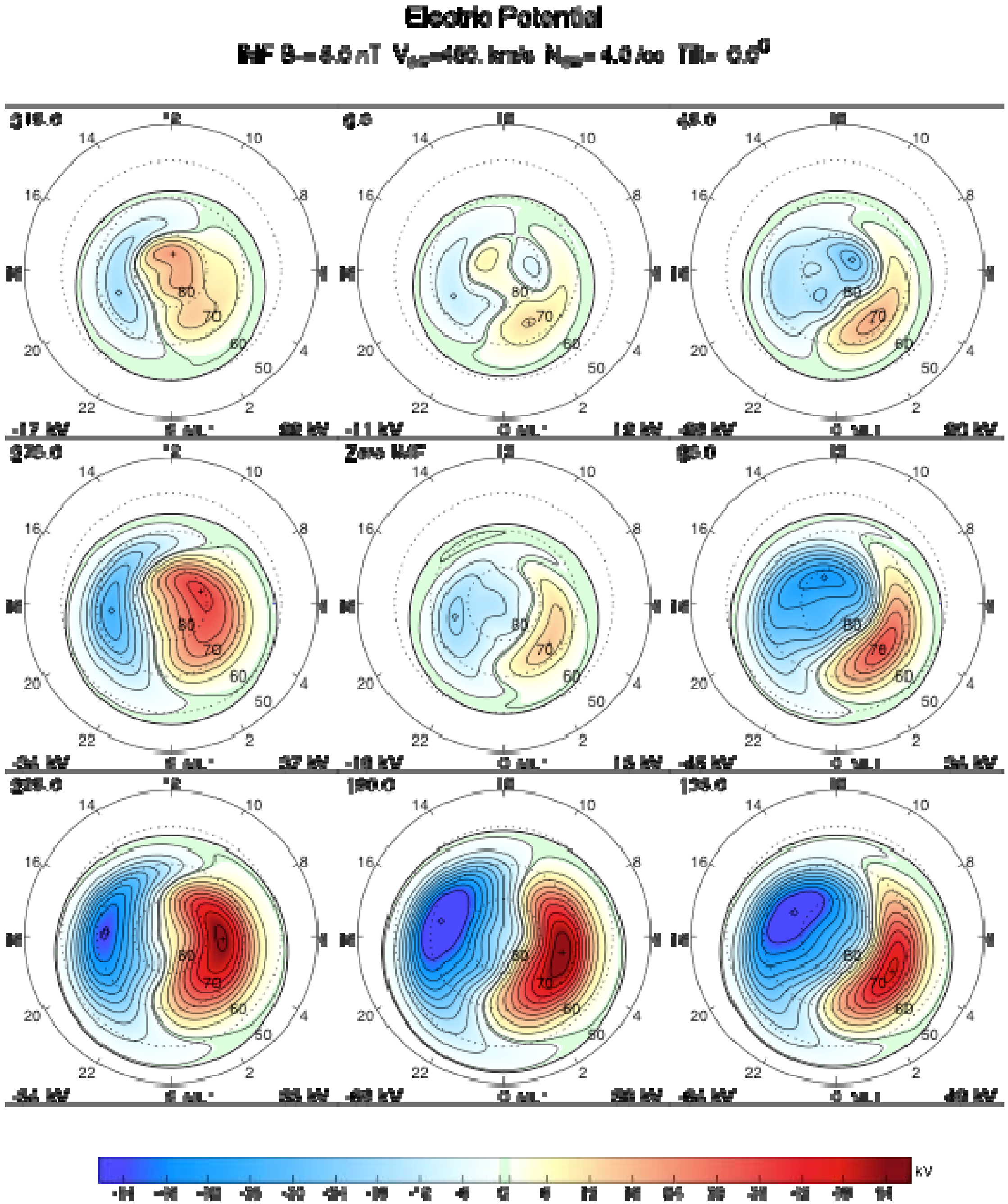
$$g_k^m = \sum_{n=0}^{4} \left(\sum_{i=0}^{5} C_{in} X_i \right) f_n(\theta)$$

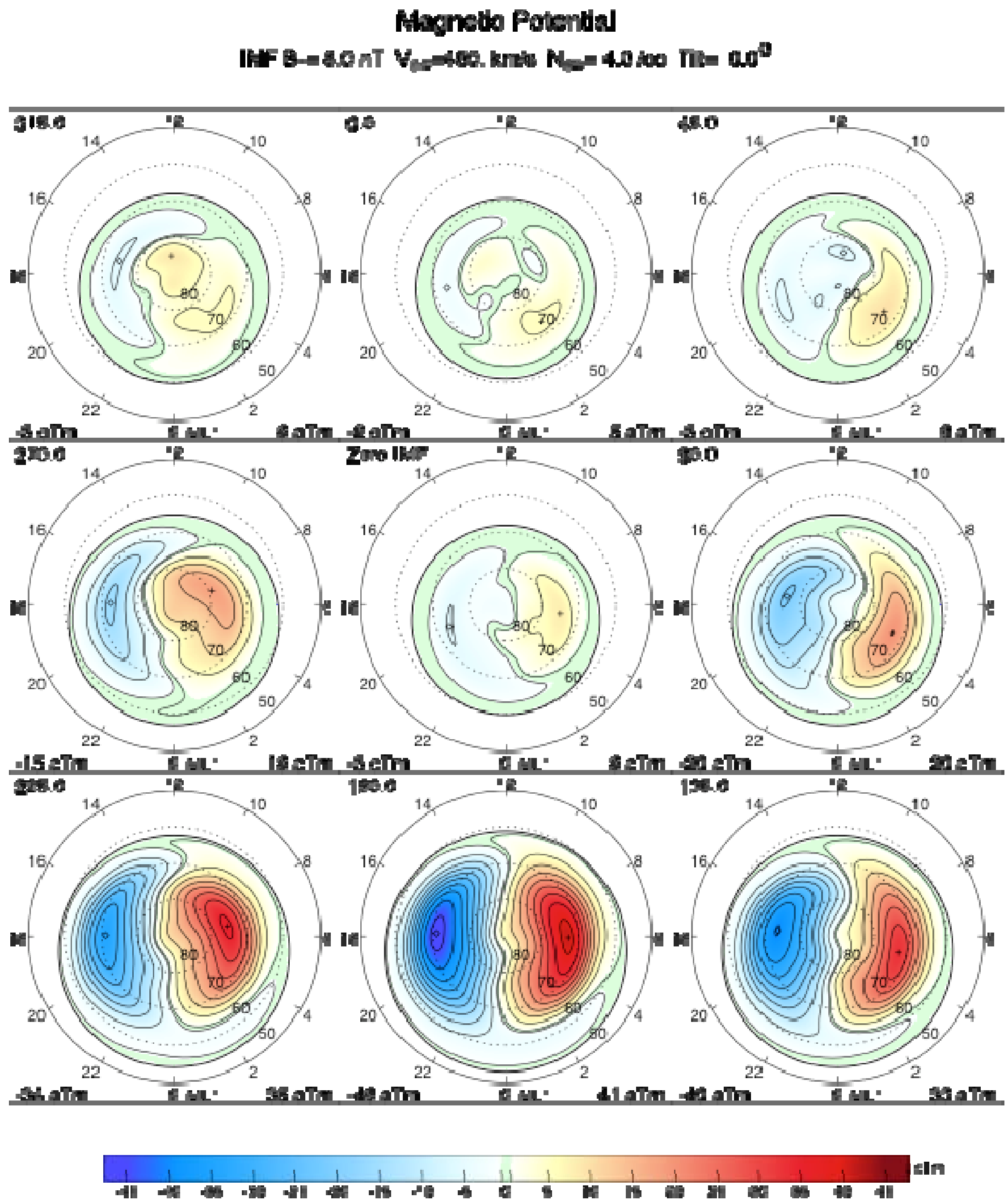
$$\begin{aligned} f_0(\theta) &= 1 \\ f_1(\theta) &= \cos(\theta) \\ f_2(\theta) &= \sin(\theta) \\ f_3(\theta) &= \cos(2\theta) \\ f_4(\theta) &= \sin(2\theta) \end{aligned}$$

$$\begin{aligned} X_0 &= 1. \\ X_1 &= E(B_T V_{SW}) \\ X_2 &= \sin(t) \\ X_3 &= \sin^2(t) \\ X_4 &= P_{SW} \\ X_5 &= AL \end{aligned}$$

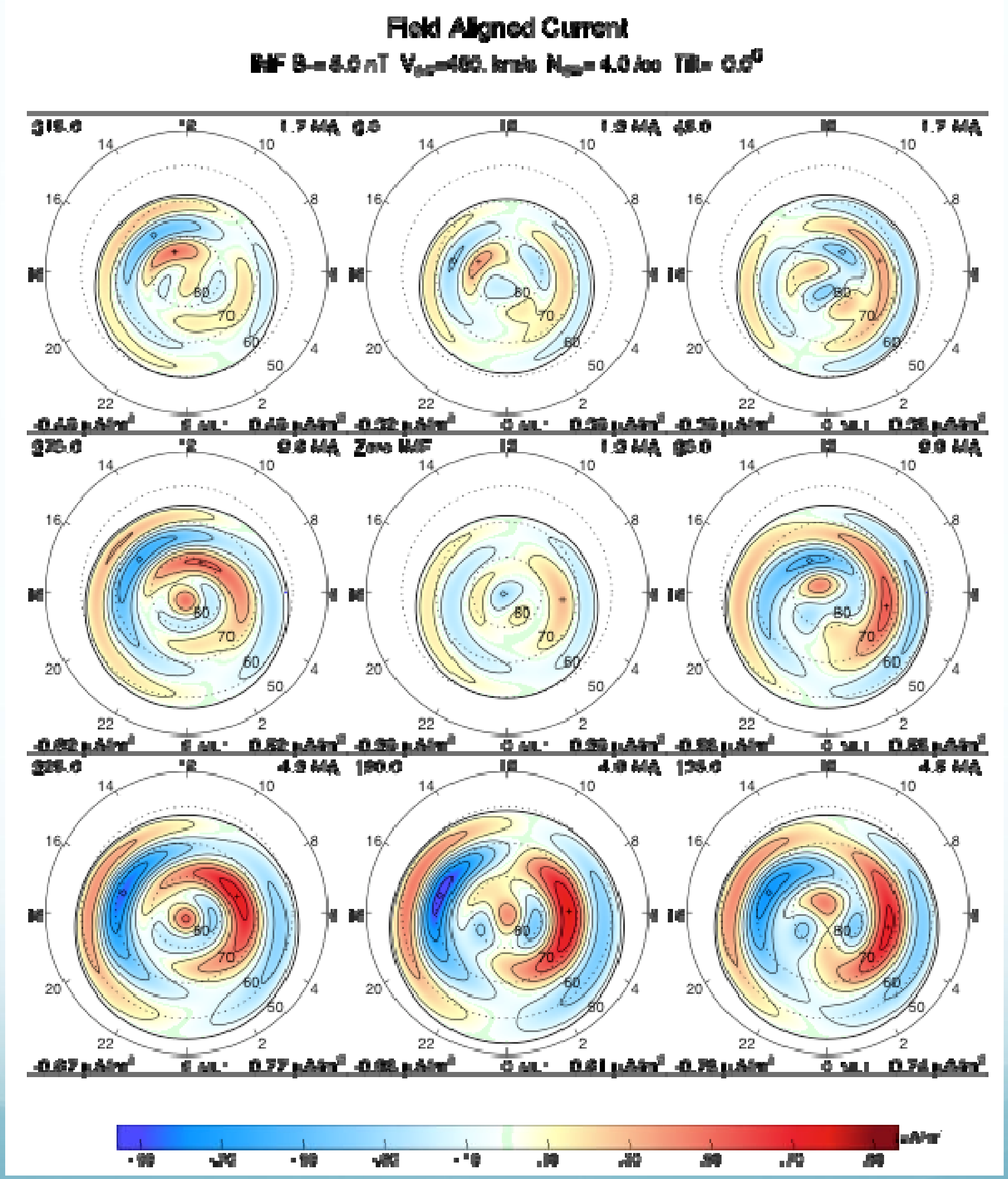
$$E(B_T V_{SW}) = \left(1 - \exp(p_1 B_T V_{SW})\right)(B_T V_{SW})^{p_2}$$

The least-square-error solution for the model's coefficients is now done in one step, without the intermediate potential patterns from sorting orbits into arbitrary bins.


Electric Potential
Zero IMF
kV

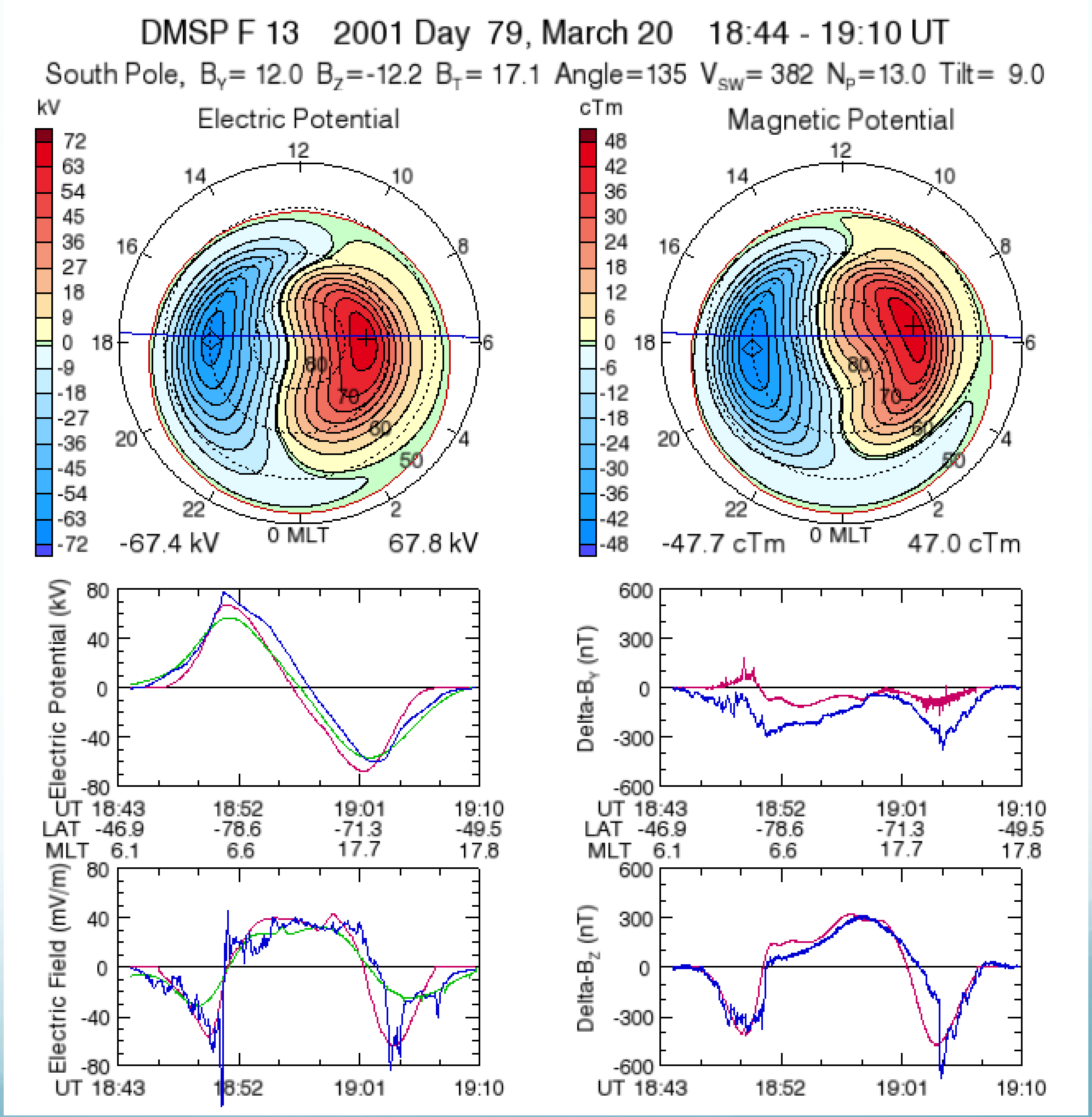


Magnetic Potential
Zero IMF
14
10
16
8
20
4
22
2
80
70
60
50



Field Aligned Current
Zero IMF

Comparisons with DMSP satellite measurements show good predictions of ionospheric electric and magnetic fields.



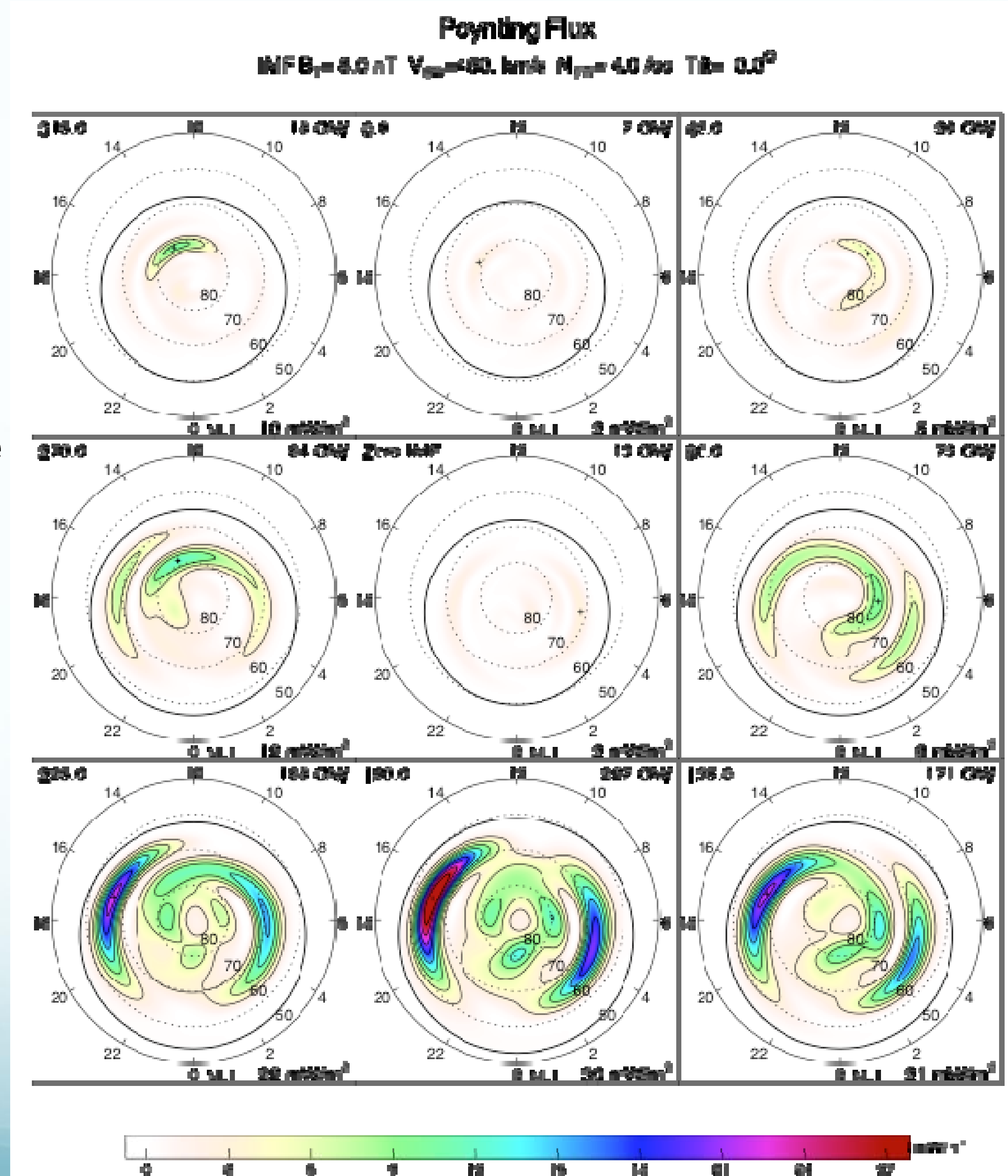
Using BOTH models, the delta-B can be used to derive the Poynting flux to the ionosphere:

$$\mathbf{S} = \mathbf{E} \times \Delta\mathbf{B} / \mu_o$$

Or the ionospheric "potential current" may be obtained from the gradient of the magnetic potential:

$$\mathbf{J}_P = \nabla_S \psi / \mu_o$$

The dot product of this $\mathbf{J}_P$ with the electric field (Joule heating) is found to be mathematically equivalent to the Poynting flux.



Predictions of Geomagnetic Perturbations

Start with the equation for the FAC: $J_{\|} = \nabla_S^2 \psi / \mu_o$

Leads to ionospheric "potential current": $\mathbf{J}_P = \nabla_S \psi / \mu_o$

If there are no conductivity gradients then: $\Sigma_{\mathrm{P}} \mathbf{E} = \nabla_{\mathrm{S}} \psi / \mu_o$

Assume fixed Hall/Pedersen conductivity ratio: $\alpha = \Sigma_{\mathrm{H}} / \Sigma_{\mathrm{P}}$

Then the equivalent/Hall current is:

$$\mathbf{J}_{\mathrm{e}} = \hat{\mathbf{B}} \times \Sigma_{\mathrm{H}} \mathbf{E} = \hat{\mathbf{B}} \times \alpha \Sigma_{\mathrm{P}} \mathbf{E} = \hat{\mathbf{B}} \times \alpha \nabla_{\mathrm{S}} \psi / \mu_o$$

With conductivity gradients: $\Sigma_{\mathrm{P}} \approx \dfrac{\mathbf{J}_p \cdot \mathbf{E}}{|E|^2}$ $\quad \Sigma_{\mathrm{P}} \mathbf{E} = (\nabla_{\mathrm{S}} \psi / \mu_o \cdot \hat{\mathbf{E}}) \hat{\mathbf{E}}$

$$\mathbf{J}_{\mathrm{e}} \approx \hat{\mathbf{B}} \times \Sigma_{\mathrm{H}} \mathbf{E} \approx \hat{\mathbf{B}} \times \alpha (\nabla_{\mathrm{S}} \psi / \mu_o \cdot \hat{\mathbf{E}}) \hat{\mathbf{E}}$$

Example of geomagnetic “prediction” using the IMF.

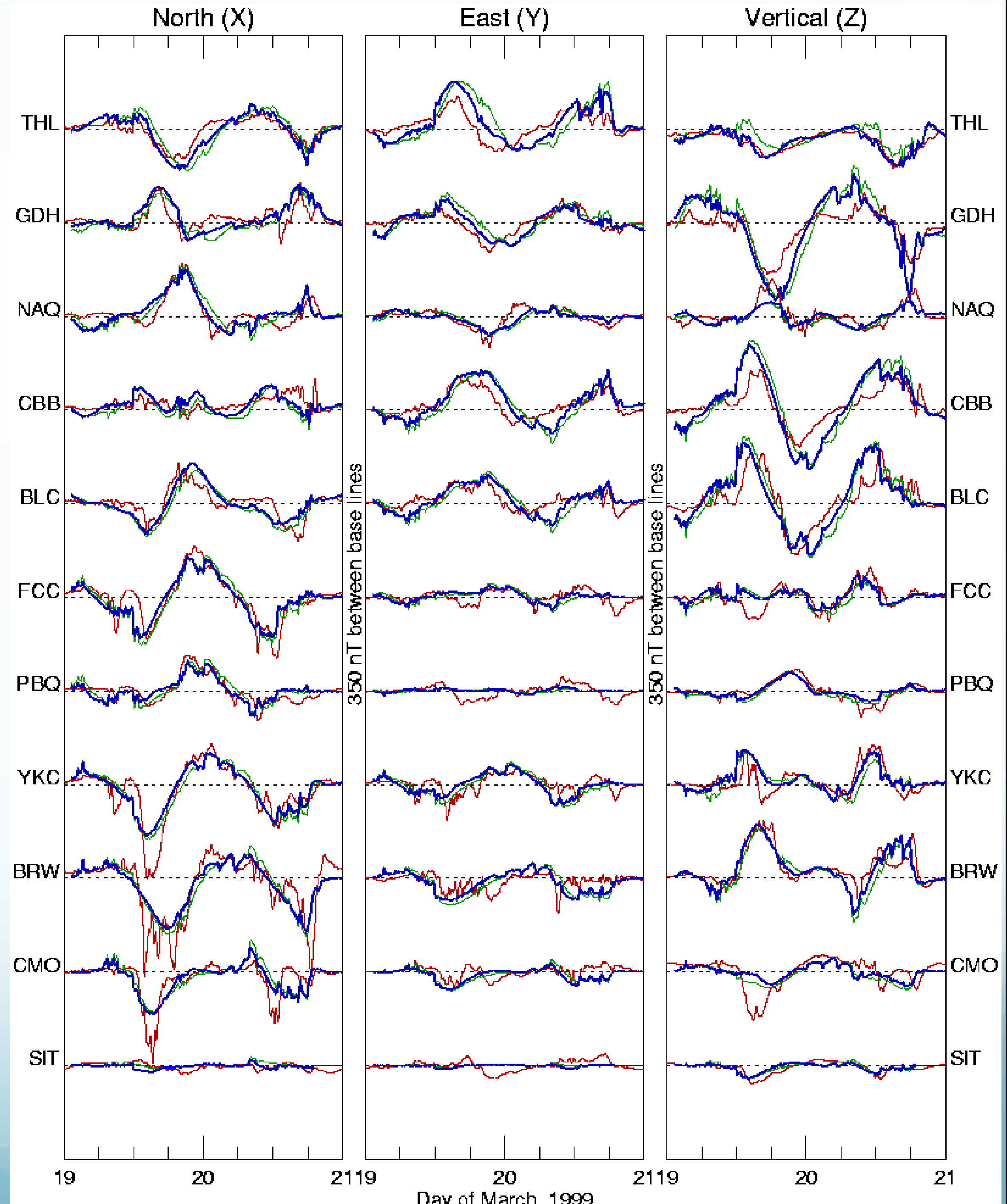
The time period is the same as for the original SEC/GEM “challenge”.

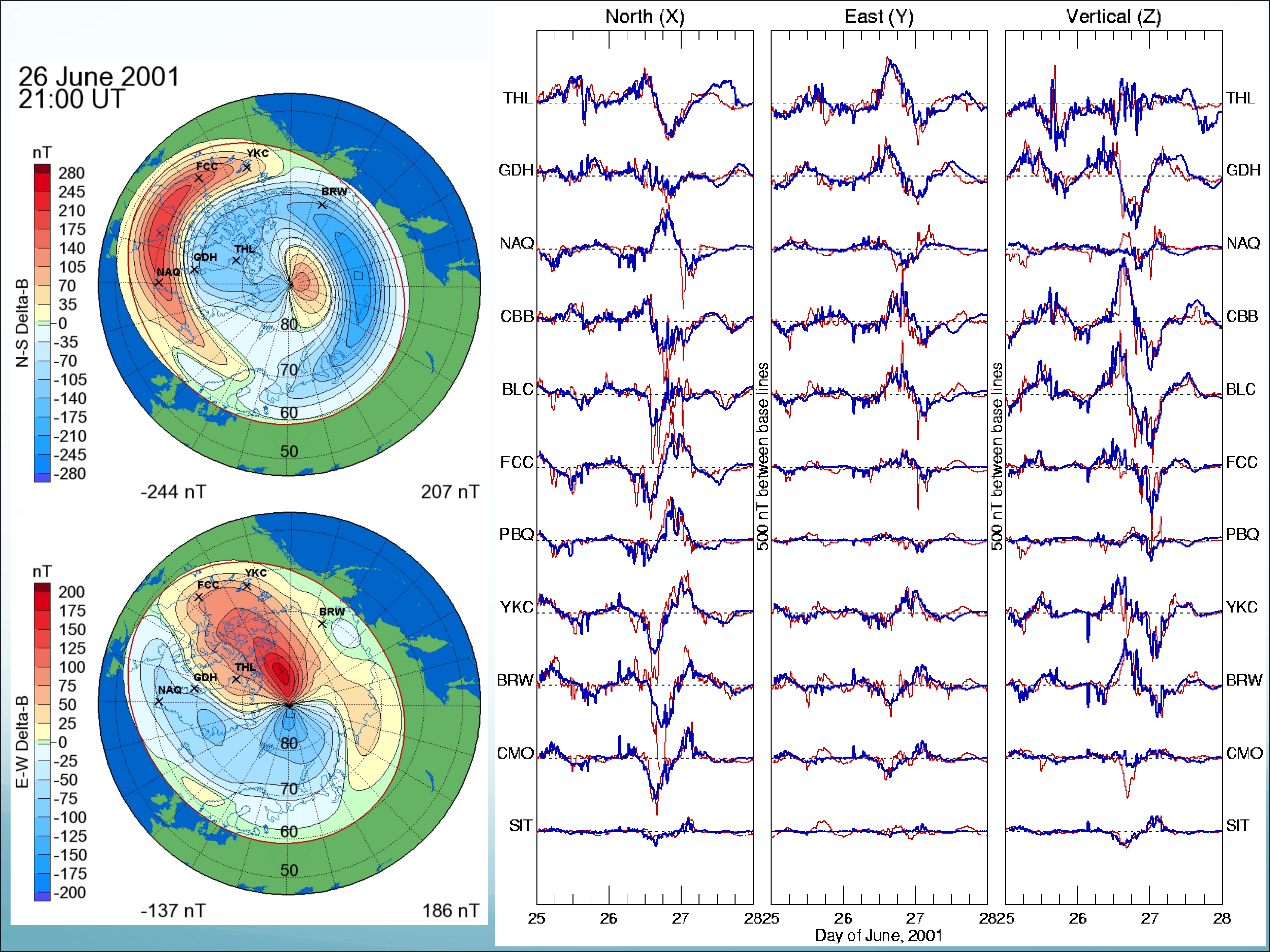
Red=actual measurements

Green=no gradient assumption

Blue=including electric field.

Uses $\mathbf{J}_e$ in a Biot-Savart integration. The *Champman and Bartels* [1940] spherical harmonic formulations work as well.





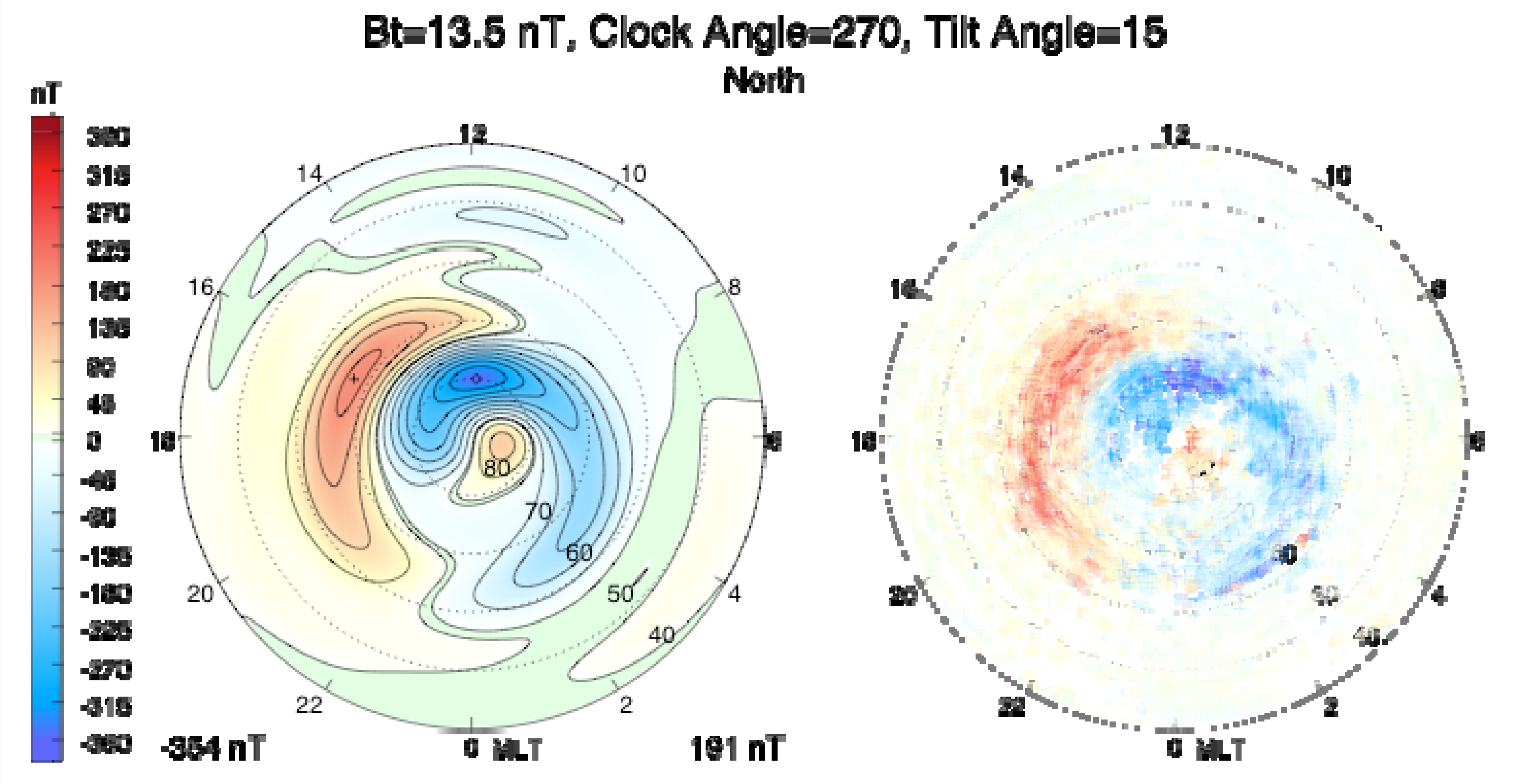
26 June 2001
21:00 UT
nT
N-S Delta-B
280
245
210
175
140
105
70
35
0
-35
-70
-105
-140
-175
-210
-245
-280
-244 nT
207 nT
E-W Delta-B
200
175
150
125
100
75
50
25
0
-25
-50
-75
-100
-125
-150
-175
-200
-137 nT
186 nT
FCC
YKC
BRW
THL
GDH
NAQ
80
70
60
50
North (X)
East (Y)
Vertical (Z)
THL
GDH
NAQ
CBB
BLC
FCC
PBQ
YKC
BRW
CMO
SIT
500 nT between base lines
25
26
27
28
Day of June, 2001

Main weakness of using the ionospheric, magnetic potential/FAC model to predict ground-level geomagnetic variations is the need to assume a fixed conductivity ratio. Causes an under-prediction in some regions. Effects of underground, induced currents are also uncertain.

Q: How can geomagnetic predictions be improved?

A: Use new empirical model based entirely on ground-level magnetometer measurements and IMF. Effects of conductivity variations and induced currents implicitly included.

Development of such a model has recently started, and is still in progress.



Delta-B North fit with Spherical Cap Harmonic Analysis (SCHA):

$$\psi(\Lambda,\varphi) = \sum_{k=0}^{15} \sum_{m=0}^{3<k} P_{n_k(m)}^{m}(\cos\Lambda)(g_k^m \cos m\varphi + h_k^m \sin m\varphi)$$

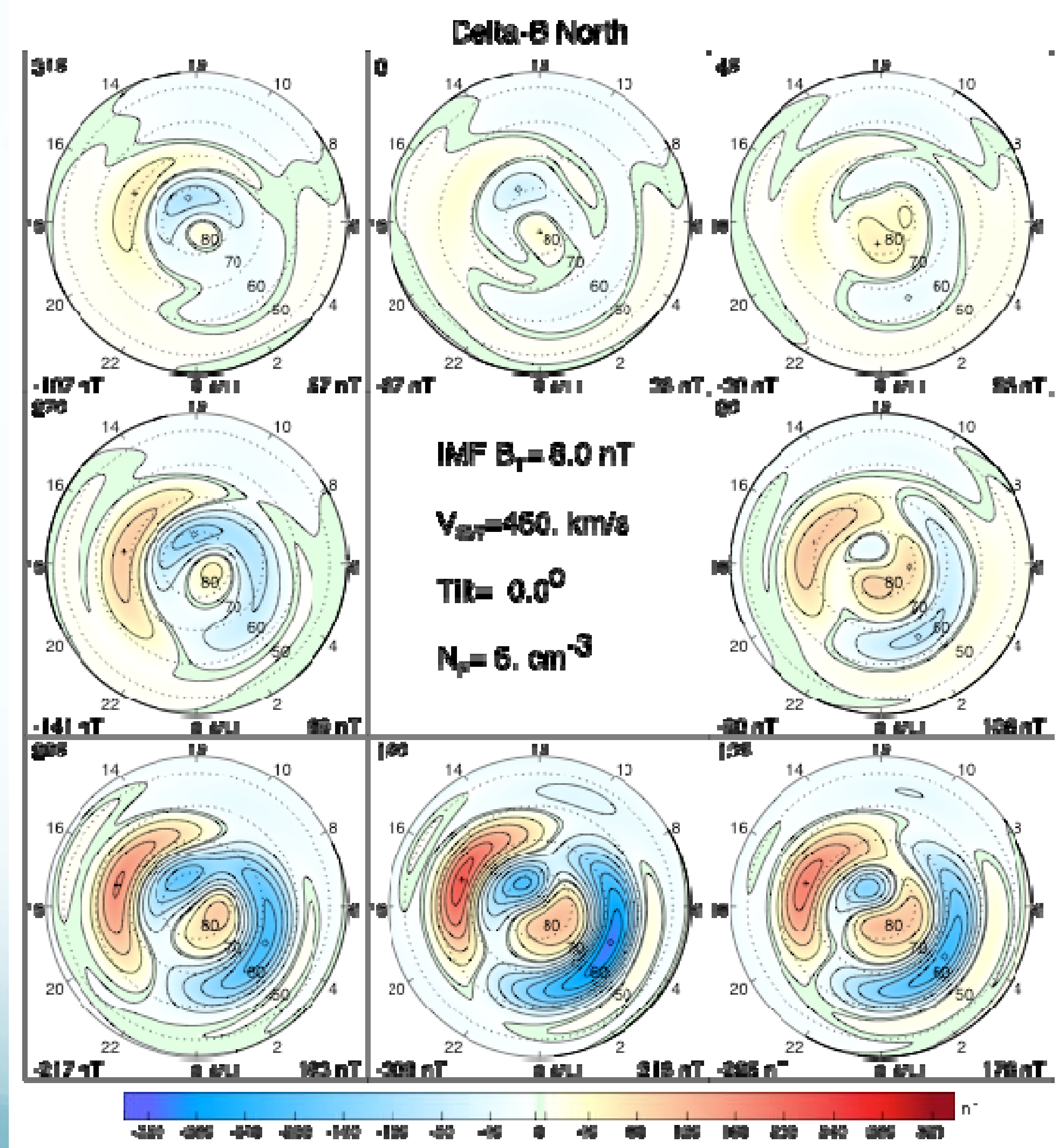
Test data from 104 magnetometer stations at over 150 times (5-min. ave.) with similar IMF conditions (from four-year interval).

New model fit from all data:

$$g_k^m = c_0 + c_1 B_T + c_2 V_{SW} + c_3 \sin(t) + c_4 P_{SW} +$$
$$c_5 B_T \cos(\theta) + c_6 V_{SW} \cos(\theta) + c_7 \sin(t)\cos(\theta) + c_8 P_{SW} \cos(\theta) + c_9 B_T \cos(2\theta) +$$
$$c_{10} B_T \sin(\theta) + c_{11} V_{SW} \sin(\theta) + c_{12} \sin(t)\sin(\theta) + c_{13} P_{SW} \sin(\theta) + c_{14} B_T \sin(2\theta)$$

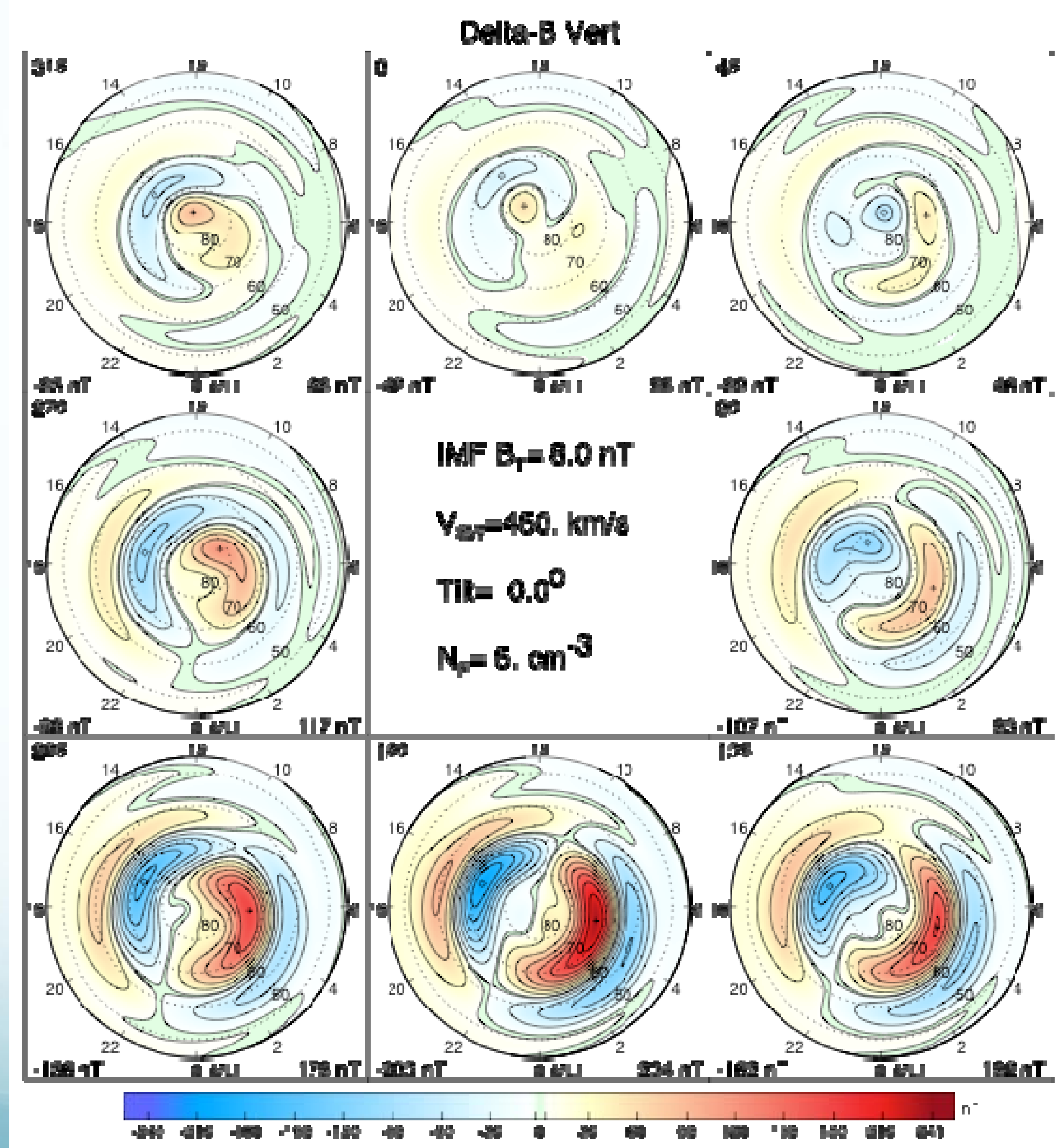
Example of model results for Northward delta-B, with 8 nT IMF at eight clock angle orientations.

Results are consistent with maps of electric potentials.



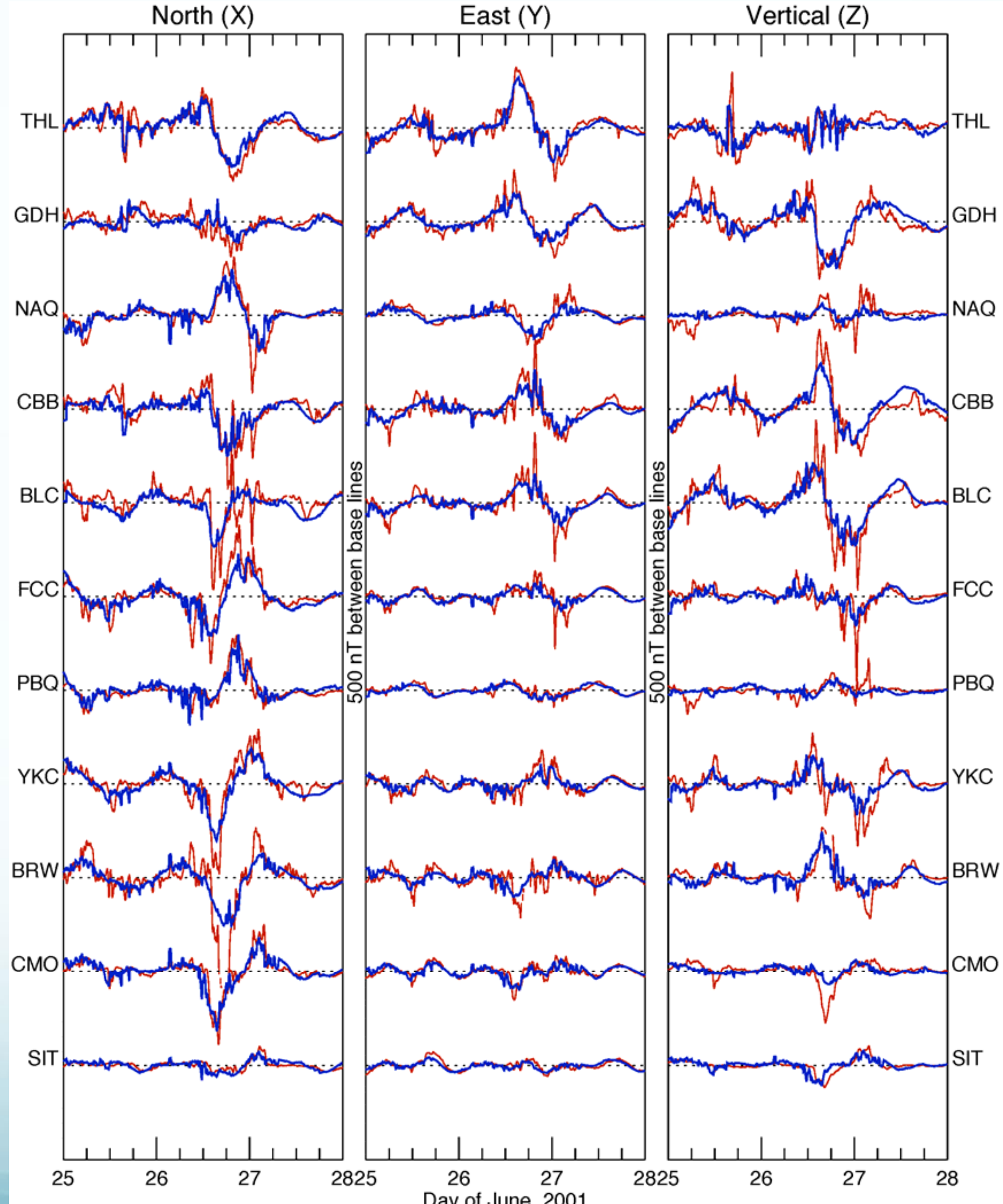
Example of model results for Vertical delta-B, with 8 nT IMF at eight clock angle orientations.

Results similar to field-aligned current maps.



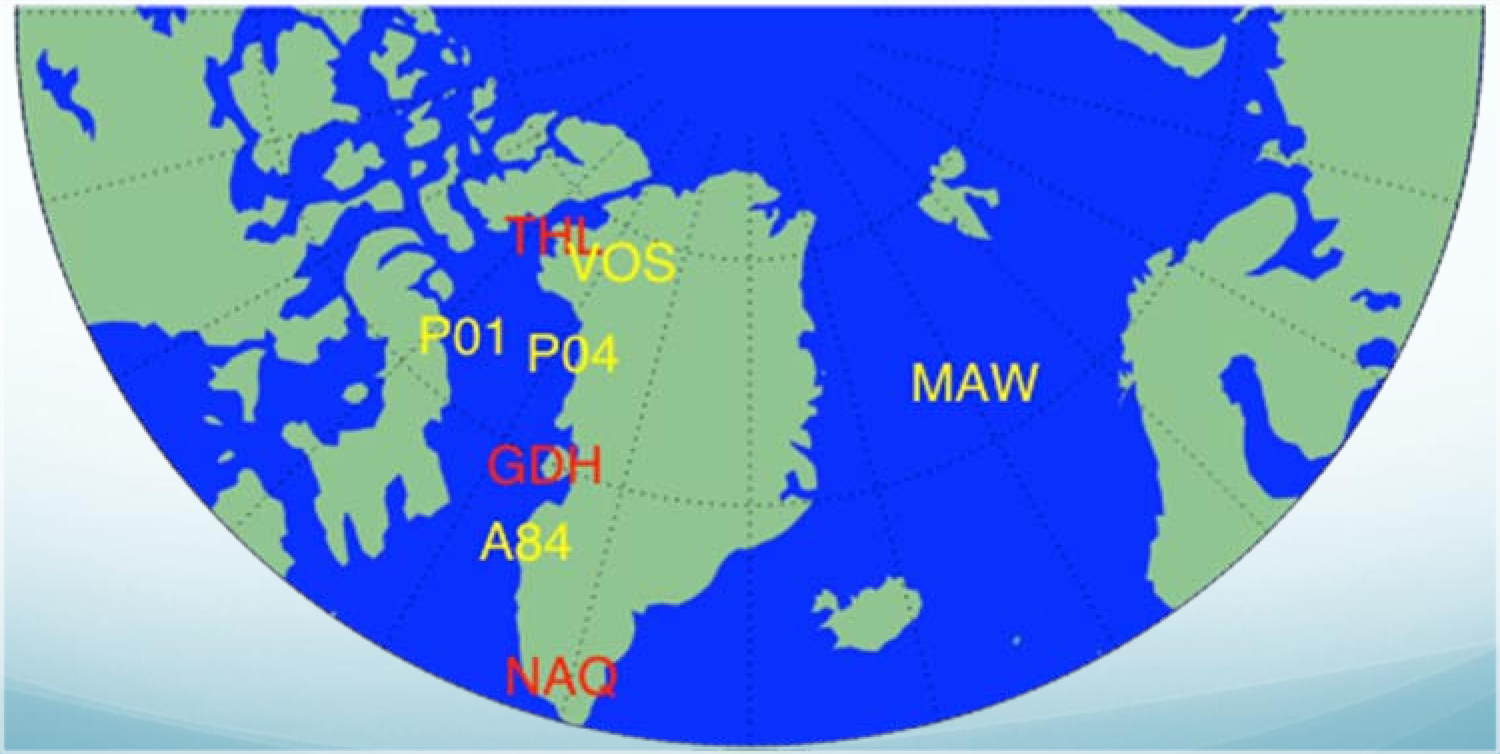
New (interim) model shows improved predictions, particular in polar cap and sub-auroral latitudes.

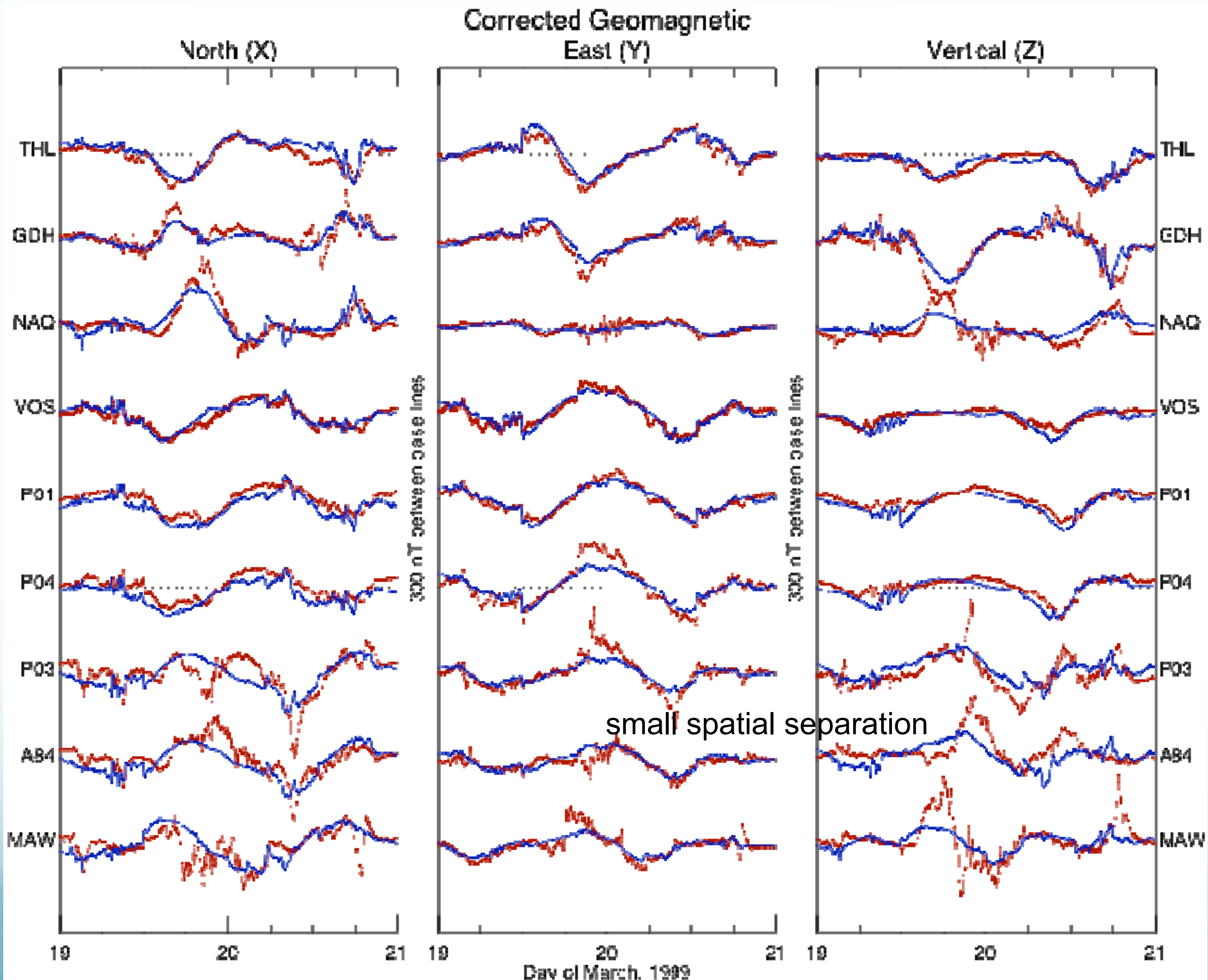
Still does not capture some variations, such as substorms.



Also tested at Southern hemisphere sites, using reversed tilt angle and IMF By inputs. Figure shows locations for 1999 example, with South pole sites (yellow) mapped to CGM conjugate locations.

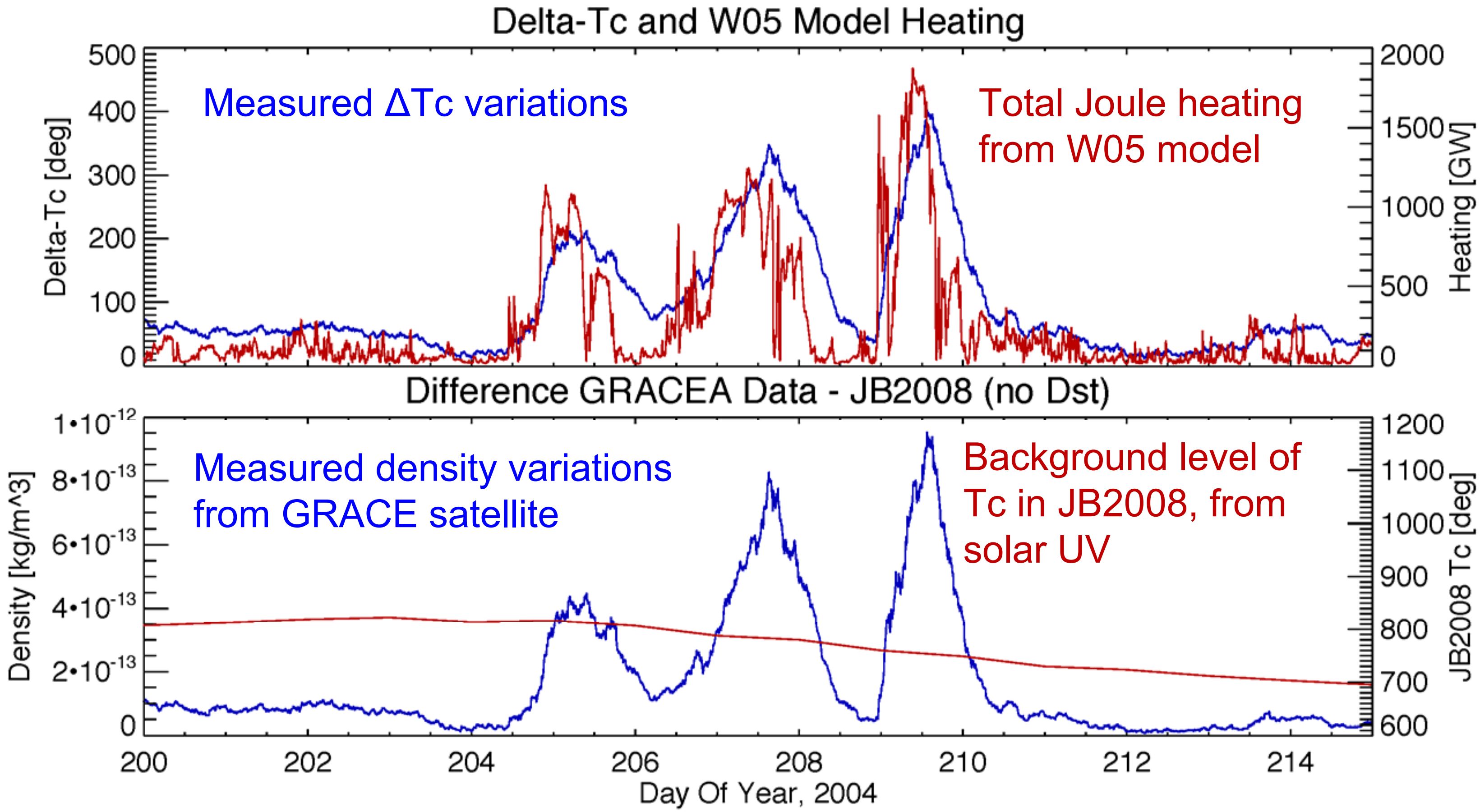
An additional site, P03, is only 3.5° East from A84 at nearly same latitude.

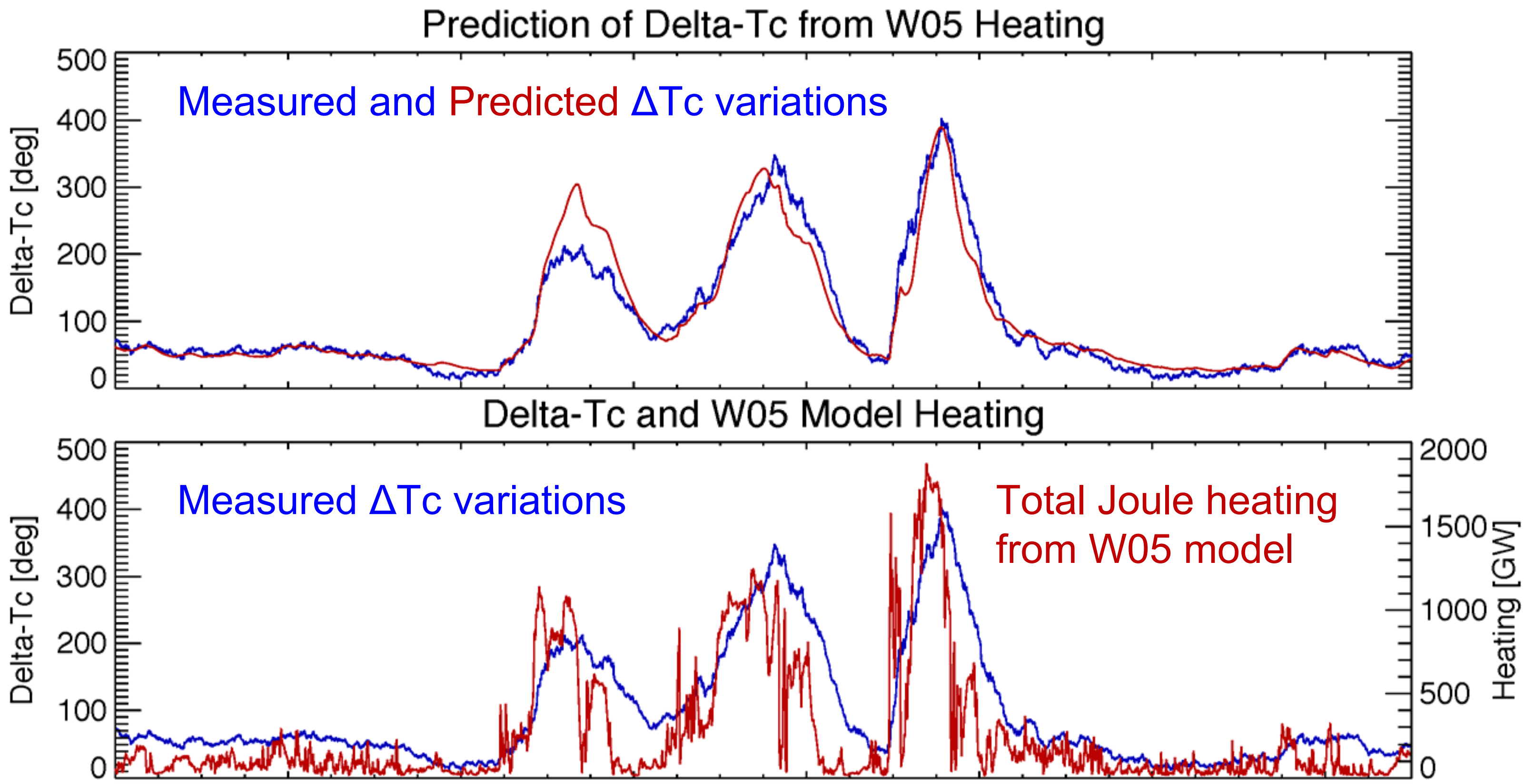



Corrected Geomagnetic
North (X)
East (Y)
Vertical (Z)
THL
GDH
NAQ
VOS
P01
P04
P03
A84
MAW
300 nT between base lines
300 nT between base lines
small spatial separation
19
20
21
19
20
21
19
20
21
Day of March, 1999

Newest effort is prediction of thermospheric densities, using Joule heating from W05 model

JB2008 = Jacchia-Bowman 2008 atmospheric density model
Tc="global nighttime minimum exospheric temperature"

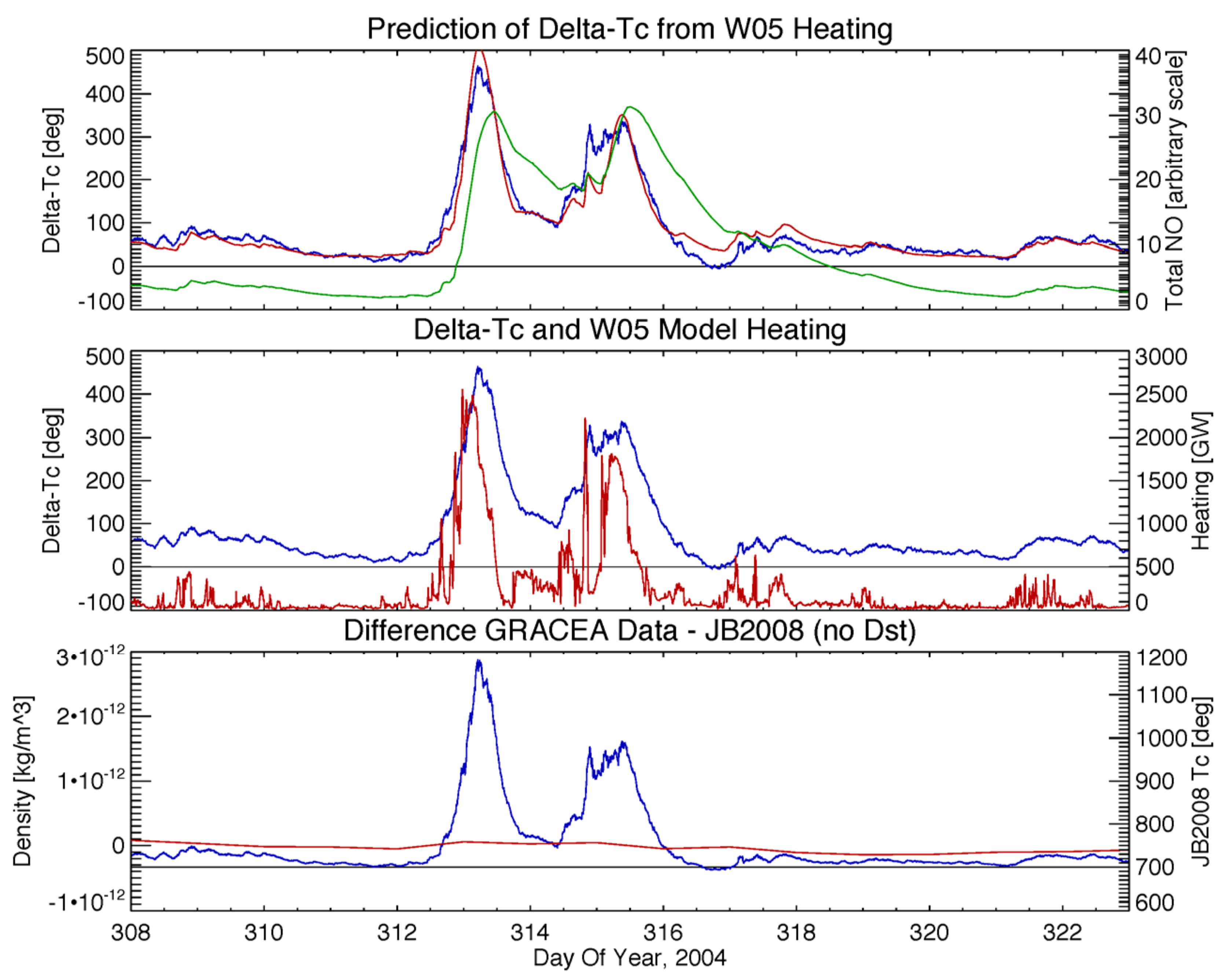


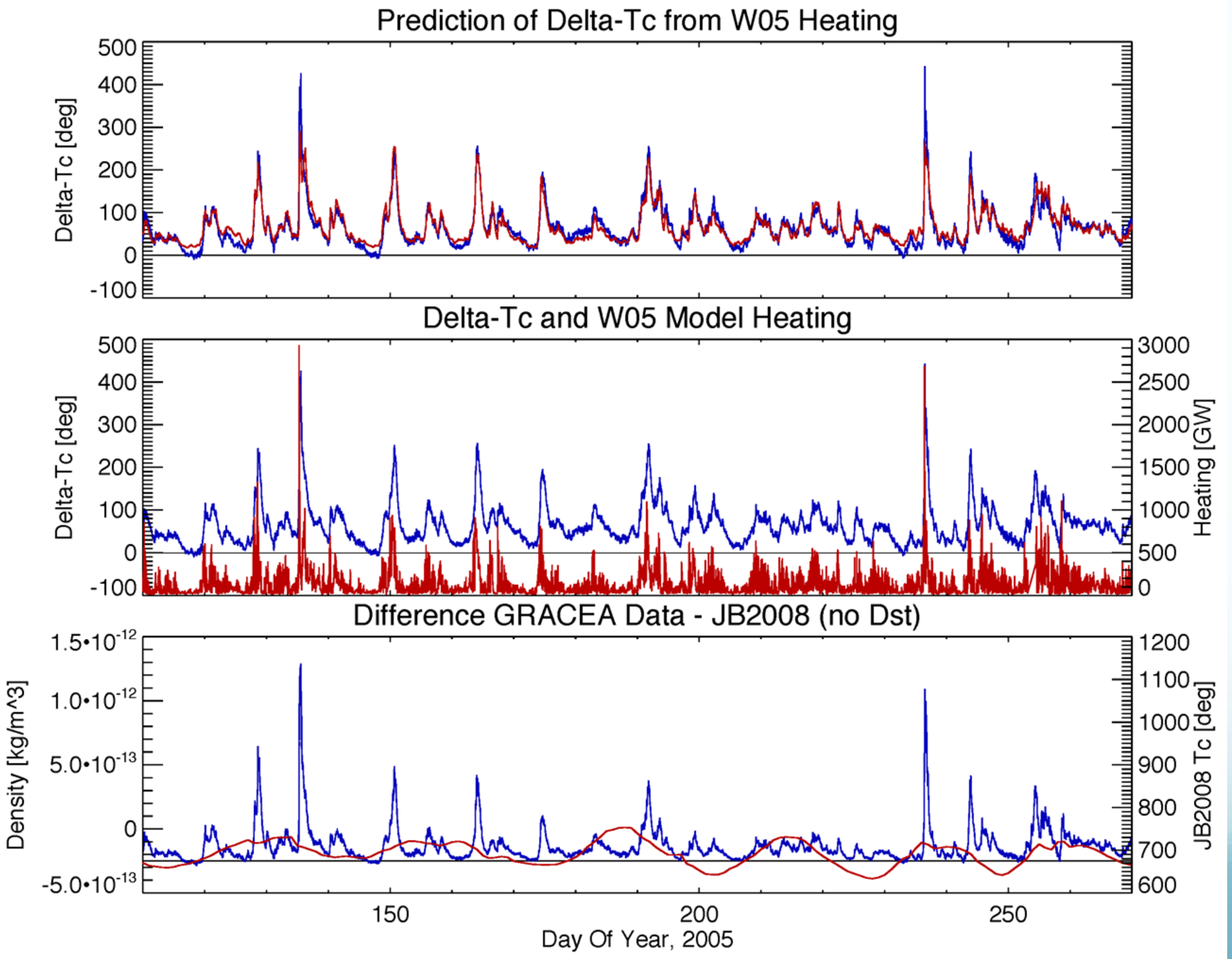


$$\Delta T_c(t_{n+1}) = \Delta T_c(t_n)(1 - \Delta t / \tau_c) + \beta H_J \Delta t$$

$$\tau_c = 13.5 - 0.28 NO \text{ (hours)}$$

$$NO(t_{n+1}) = NO(t_n)(1 - \Delta t / \tau_{NO}) + \gamma H_J \Delta t$$


Prediction of Delta-Tc from W05 Heating
Delta-Tc [deg]
Total NO [arbitrary scale]
Delta-Tc and W05 Model Heating
Delta-Tc [deg]
Heating [GW]
Difference GRACEA Data - JB2008 (no Dst)
Density [kg/m^3]
JB2008 Tc [deg]
Day Of Year, 2004


Prediction of Delta-Tc from W05 Heating
Delta-Tc [deg]
500
400
300
200
100
0
-100
Delta-Tc and W05 Model Heating
Delta-Tc [deg]
Heating [GW]
3000
2500
2000
1500
1000
500
0
Difference GRACEA Data - JB2008 (no Dst)
Density [kg/m^3]
1.5•10^-12
1.0•10^-12
5.0•10^-13
0
-5.0•10^-13
JB2008 Tc [deg]
1200
1100
1000
900
800
700
600
150
200
250
Day Of Year, 2005

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