SWMF Magnetosphere

Gábor Tóth

Alex Glocer, Yingjuan Ma, Xing Meng, Dalal Najib, Yiqun Yu, Bart van der Holst, Tamas Gombosi

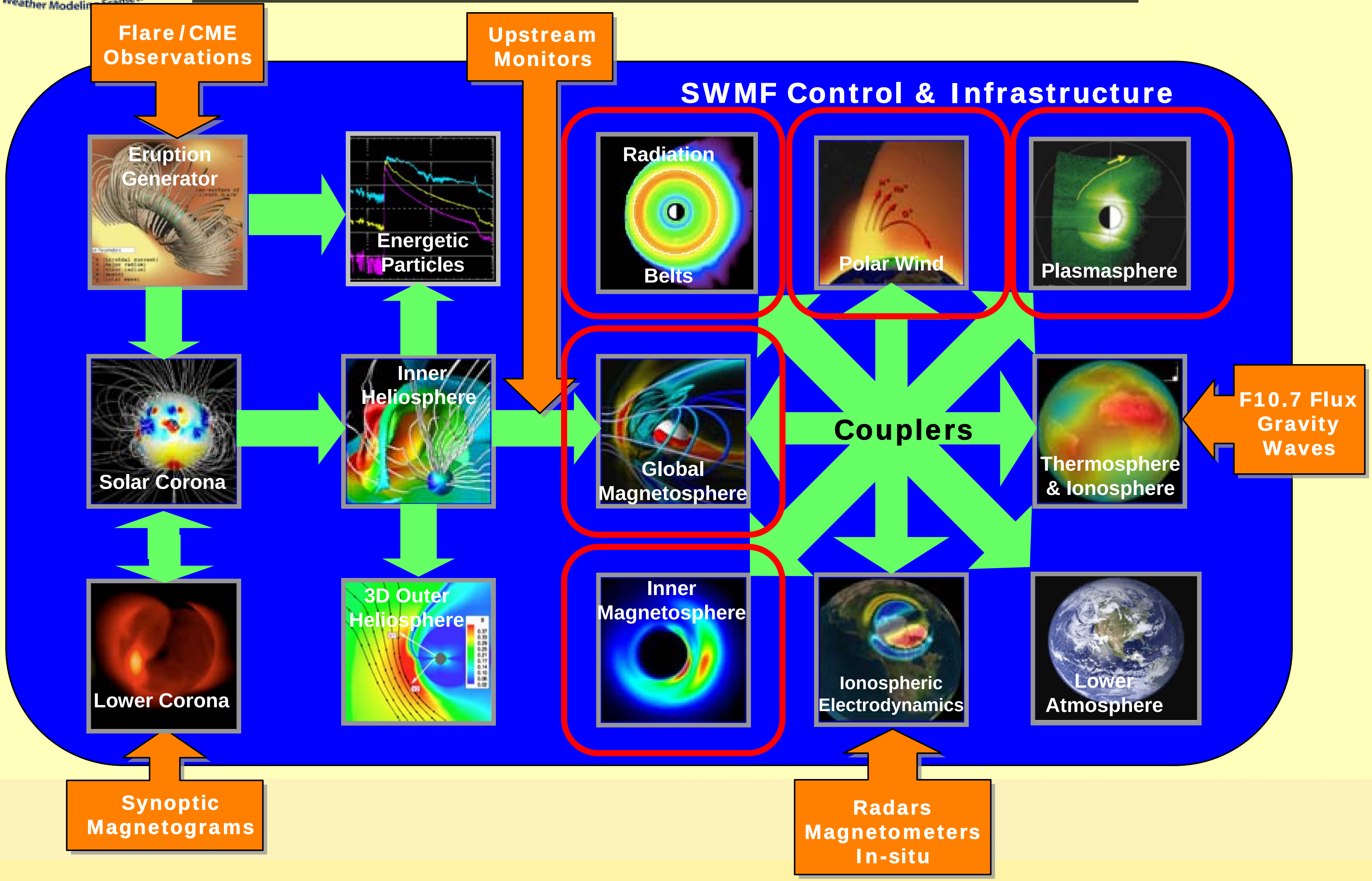
Center for Space Environment Modeling

University Of Michigan

Outline

- Magnetospheric components of the SWMF
- MHD with non-isotropic pressure
- Multi-ion MHD
- Summary

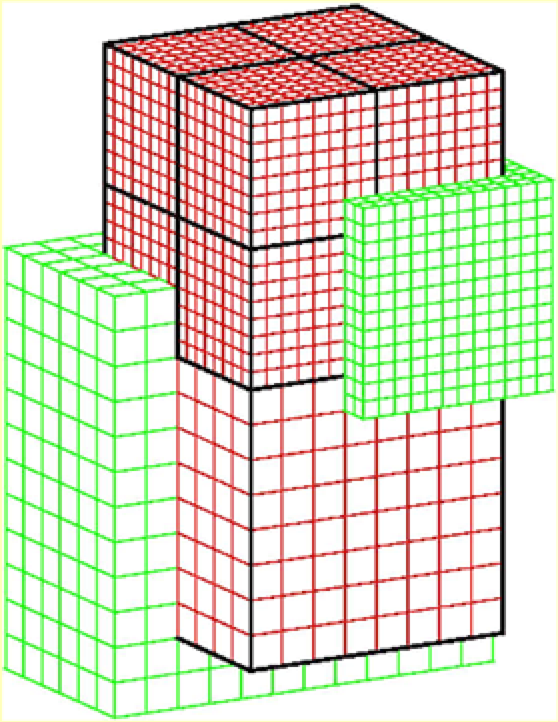
Magnetospheric Components in the Space Weather Modeling Framework



SWMF is freely available at http://csem.engin.umich.edu and via CCMC

BATS-R-US

Block Adaptive Tree Solar-wind Roe Upwind Scheme



Physics
- Classical, semi-relativistic and Hall MHD
- Multi-species, multi-fluid, anisotropic pressure
- Radiation hydrodynamics with multigroup diffusion
- Multi-material, non-ideal equation of state
- Solar wind turbulence

Numerics
- Conservative finite-volume discretization
- Parallel block-adaptive grid
- Cartesian and generalized coordinates
- Splitting the magnetic field into $B_0 + B_1$
- Divergence B control: 8-wave, CT, projection, parabolic/hyperbolic
- Shock-capturing TVD schemes: Rusanov, HLLE, AW, Roe, HLLD
- Explicit, point-implicit, semi-implicit, fully implicit time stepping

Applications
- Sun, heliosphere, magnetospheres, unmagnetized planets, moons, comets…

100,000+ lines of Fortran 90 code with MPI parallelization

Anisotropic MHD

- **What is it?**
  - Different pressures parallel and perpendicular to the magnetic field
- **Space physics applications**
  - Reconnection
  - Magnetosphere
  - Coupling with inner magnetosphere models (e.g. HEIDI, RAM, CRCM)
  - Solar wind heating
- **Difficulties**
  - New set of equations to solve
  - Physical instabilities: fire-hose, mirror, proton cyclotron
- **Combinations with more physics**
  - Separate electron pressure
  - Hall MHD, semi-relativistic, multi-ion

Resistive MHD with electrons and anisotropic ion pressure

Mass conservation: $$\frac{\partial \rho}{\partial t} + \nabla\cdot(\rho\mathbf{u}) = 0$$

Momentum: $$\frac{\partial \rho\mathbf{u}}{\partial t} + \nabla\cdot(\rho\mathbf{u}\mathbf{u} + P) = \mathbf{J}\times\mathbf{B}$$

$$P = (p_\perp + p_e)I + (p_\parallel - p_\perp)\mathbf{b}\mathbf{b} \qquad p = \frac{2p_\perp + p_\parallel}{3}$$

Induction: $$\frac{\partial \mathbf{B}}{\partial t} + \nabla\times\mathbf{E} = 0$$

Pressure: $$\frac{\partial p_\perp}{\partial t} + \nabla\cdot(p_\perp\mathbf{u}) = \frac{1}{3\tau}(p_\parallel - p_\perp) + \frac{2}{\tau_{ie}}(p_e - p) - p_\perp\nabla\cdot\mathbf{u} + p_\perp\mathbf{b}\cdot(\nabla\mathbf{u})\cdot\mathbf{b}$$

$$\frac{\partial p}{\partial t} + \nabla\cdot(p\mathbf{u}) = (\gamma-1)[-p\nabla\cdot\mathbf{u} + \eta\mathbf{J}^2] + \frac{2}{\tau_{ie}}(p_e - p)$$

$$\frac{\partial p_\parallel}{\partial t} + \nabla\cdot(p_\parallel\mathbf{u}) = \frac{2}{3\tau}(p_\perp - p_\parallel) + \frac{2}{\tau_{ie}}(p_e - p) - 2p_\parallel\mathbf{b}\cdot(\nabla\mathbf{u})\cdot\mathbf{b}$$

$$\tau_{ie} = \frac{2}{3}\frac{M_i}{\eta e^2 n_e}$$

Electron pressure:

$$\frac{\partial p_e}{\partial t} + \nabla\cdot(p_e\mathbf{u}) = (\gamma-1)\left[-p_e\nabla\cdot\mathbf{u} + \eta\mathbf{J}^2 + \nabla\cdot(\kappa\mathbf{b}\mathbf{b}\cdot\nabla T_e)\right] + \frac{2}{\tau_{ie}}(p - p_e)$$

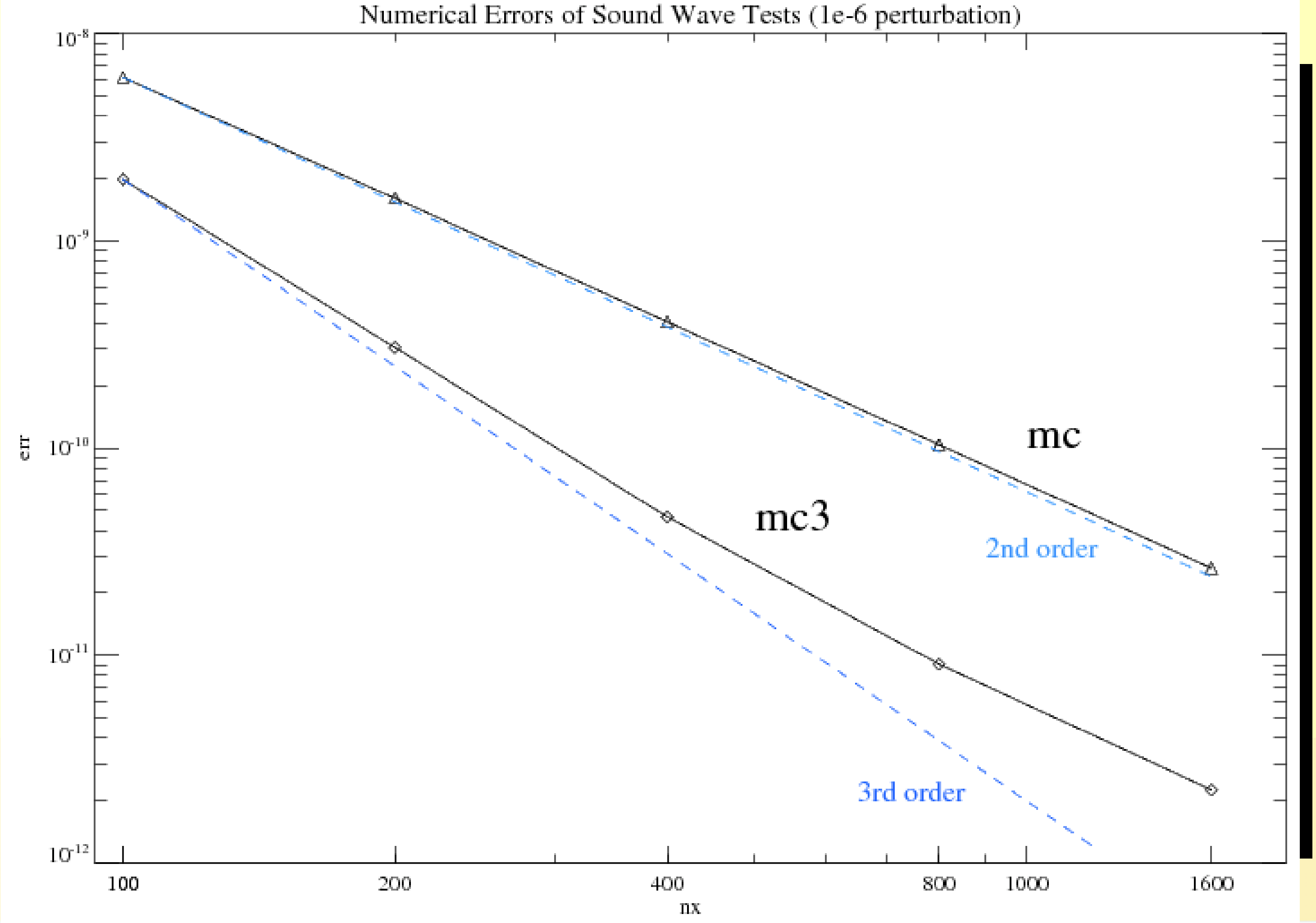
Electric field: $$\mathbf{E} = -\mathbf{u}\times\mathbf{B} + \eta\mathbf{J} \qquad \mathbf{b} = \mathbf{B}/B$$

Current: $$\mathbf{J} = \nabla\times\mathbf{B}$$

Verification tests for anisotropic pressure

Sound wave propagating parallel to magnetic field at $c_s = \sqrt{3p_\parallel/\rho}$

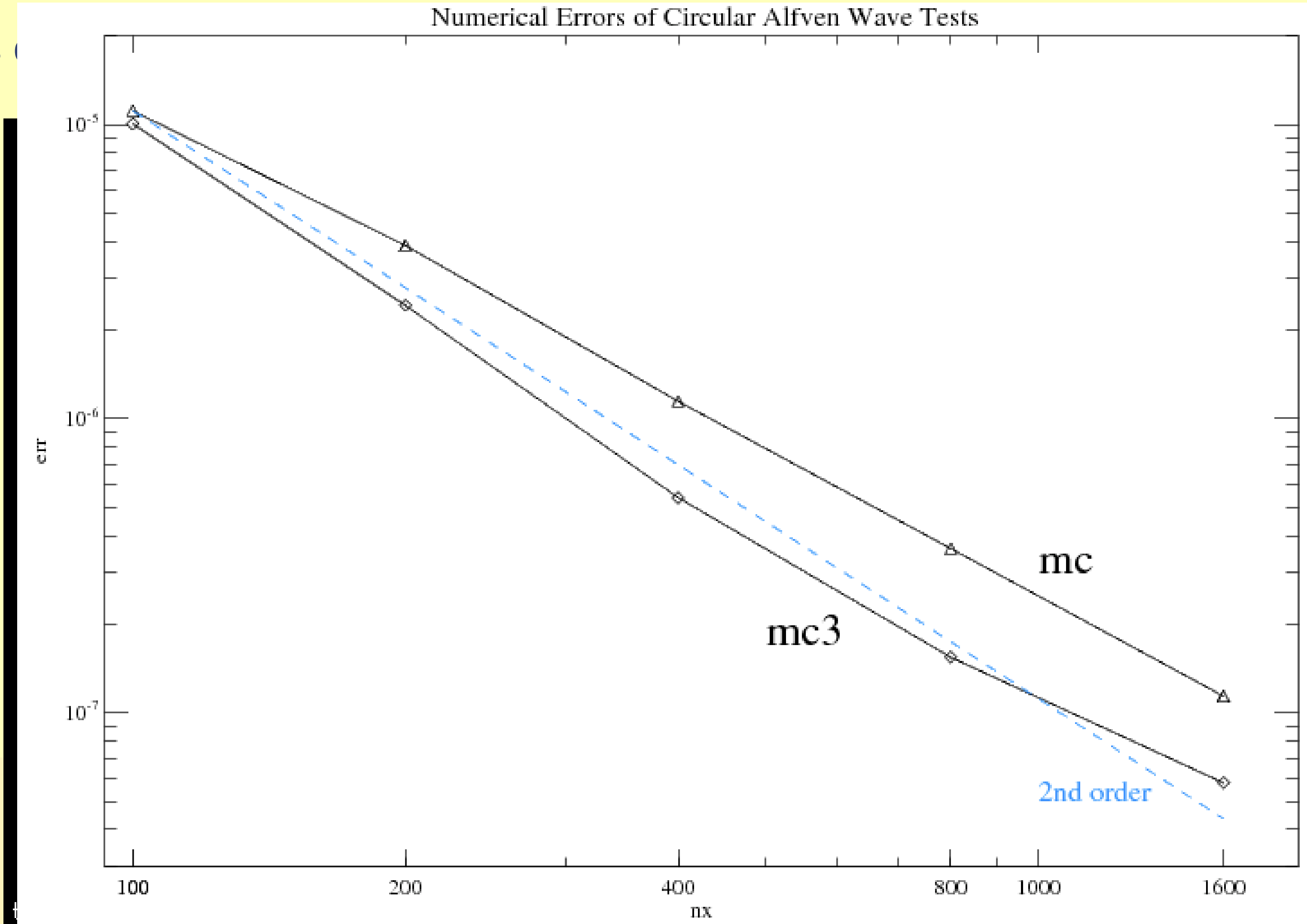
Grid convergence for smooth problem:



Verification tests for anisotropic pressure

Circularly polarized Alfven wave propagating at $v_A = \sqrt{(\mathbf{B}^2 + p_\perp - p_\parallel)/\rho}$

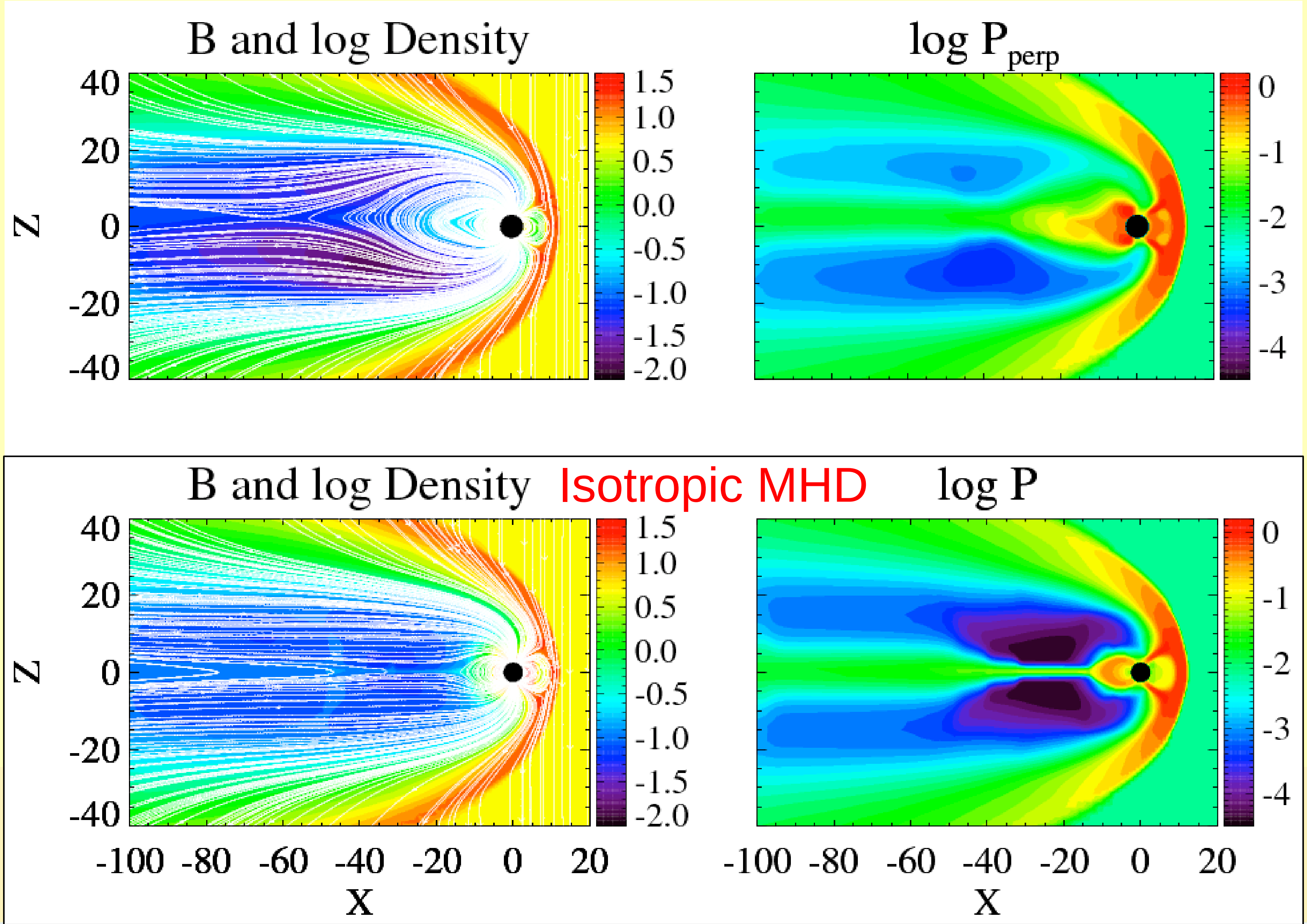
This



Limiting the Anisotropy

- **Instabilities**
  - Fire-hose: $$\frac{p_\parallel}{p_\perp} > 1 + \frac{B^2}{p_\perp}$$
  - Mirror: $$\frac{p_\perp}{p_\parallel} > 1 + \frac{B^2}{2p_\perp}$$
  - Proton cyclotron: $$\frac{p_\perp}{p_\parallel} > 1 + 0.847\left(\frac{B^2}{2p_\parallel}\right)^{0.48}$$
  - In unstable regions we make the ion pressure isotropic
- **Ion-ion, ion-electron and/or wave-ion interactions:**
  - Push ion pressure towards isotropic distribution with time rate τ

Preliminary Magnetosphere Run



Plans for Anisotropic MHD

- **Use energy equation to capture shocks**
- **Combine anisotropic MHD with separate electron equation**
- **Magnetospheric simulations**
  - Comparison with data (e.g. Cluster, Wind)
- **Combine with Hall MHD for reconnection studies**
  - GEM challenge
- **Apply to solar corona**
- **Publish papers …**

Multi-Fluid MHD

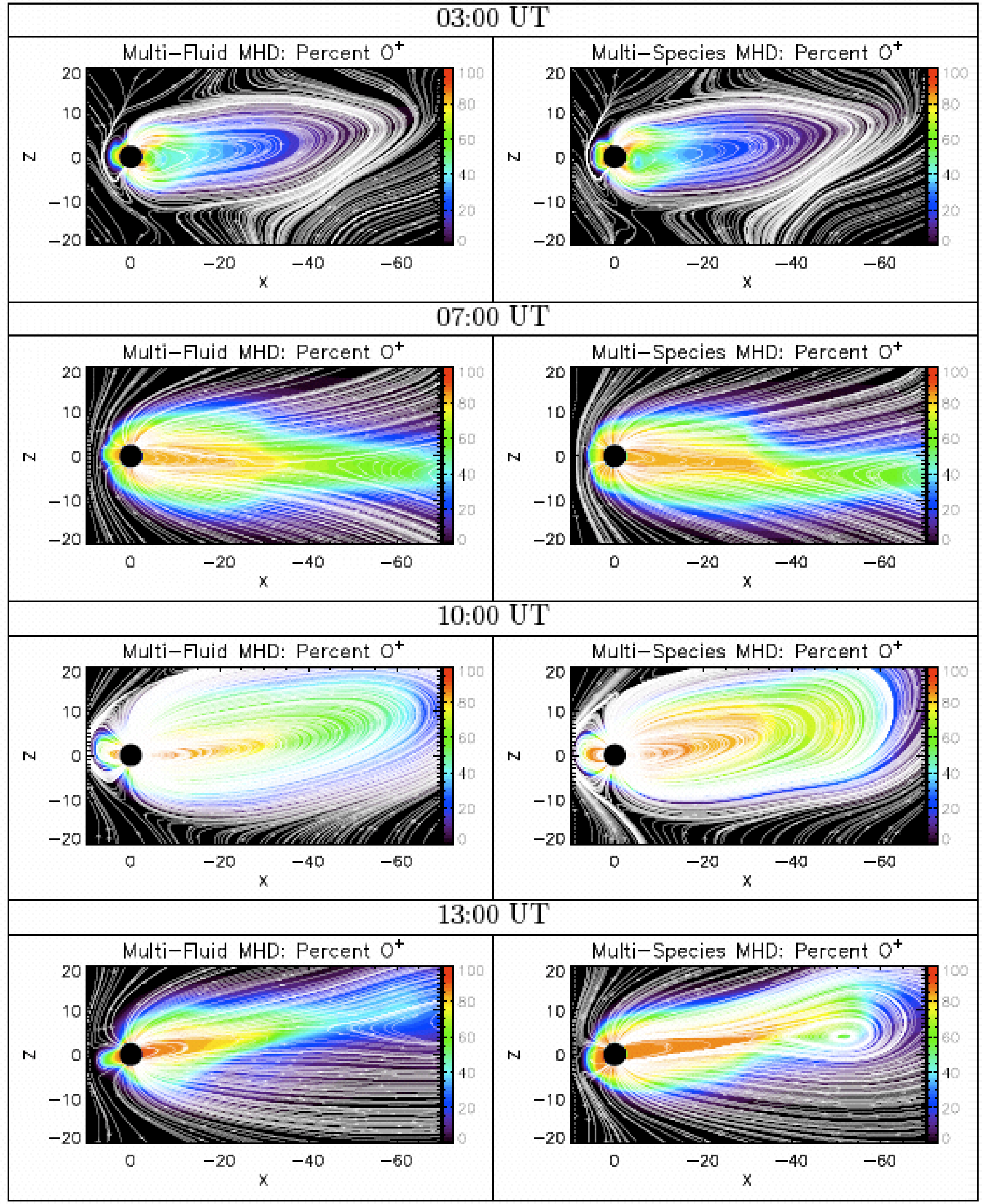
- **Each fluid has separate densities, velocities and temperatures.**
- **Multi-Fluid MHD has many space physics applications**
  - ionospheric outflow: coupling with PWOM
  - Earth magnetosphere
  - Martian ionosphere
  - Outer Heliosphere interaction with interstellar medium
- **Fluids are coupled by collisions, charge exchange and chemical reactions.**
- **BATS-R-US now contains a general multi-fluid solver with arbitrary number of ion and neutral fluids.**

Initial Results
(Glocer et al, 2009, JGR)

- **Modeling two magnetic storms**
  - May 4, 1998
  - March 31, 2001
- **Multi-fluid BATS-R-US running in the SWMF coupled with**
  - Polar Wind Outflow Model
  - Ridley Ionosphere-electrodynamics Model
  - Rice Convection Model (inner magnetosphere)
- **Comparison with**
  - single fluid model
  - global indexes (Dst, CPCP)
  - in situ satellite measurements

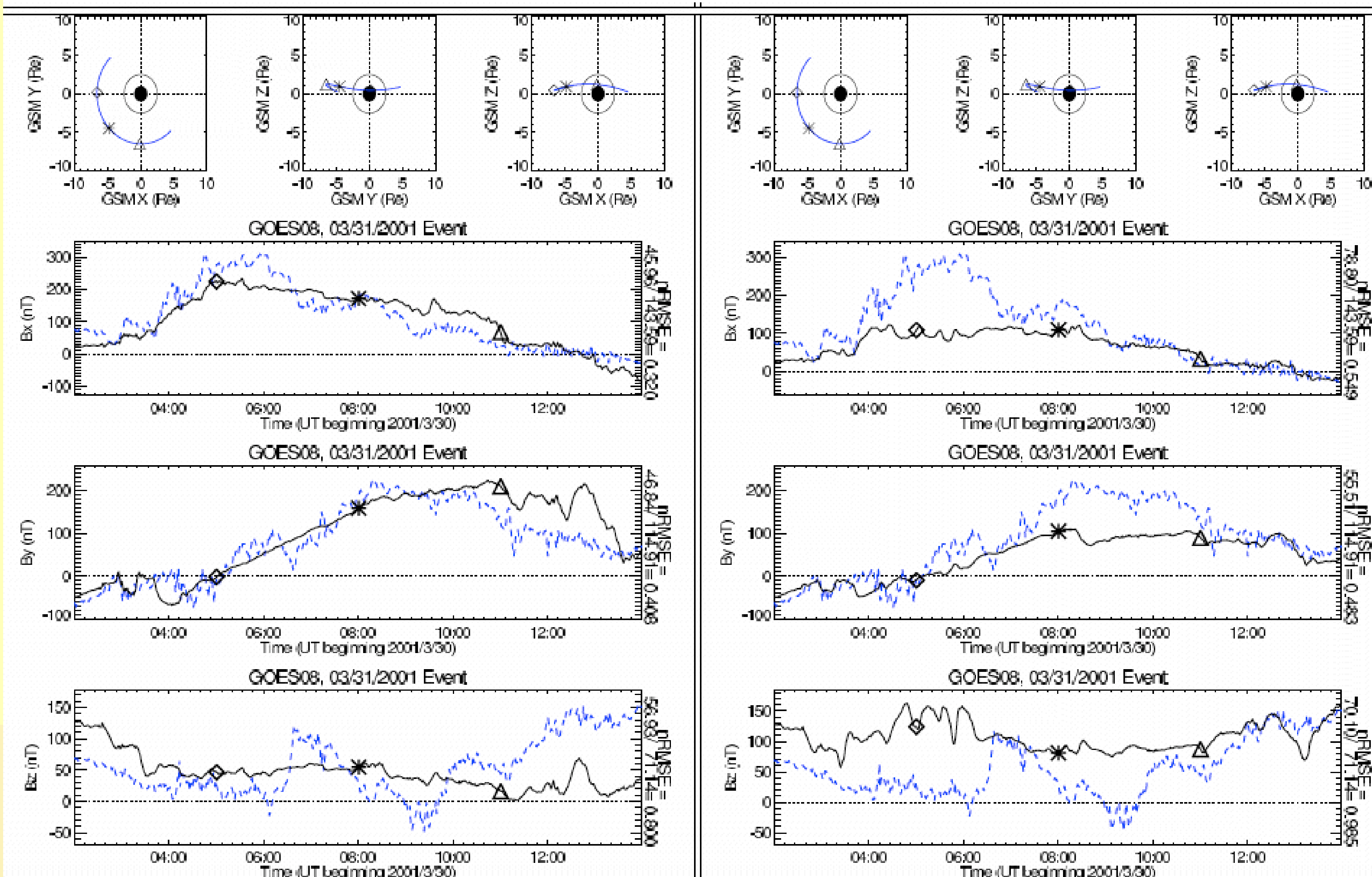
O^+/H^+ Ratio for March 31 Storm

- **Multi-Fluid vs. Multi-species**
  - Similar near Earth
  - Different further away

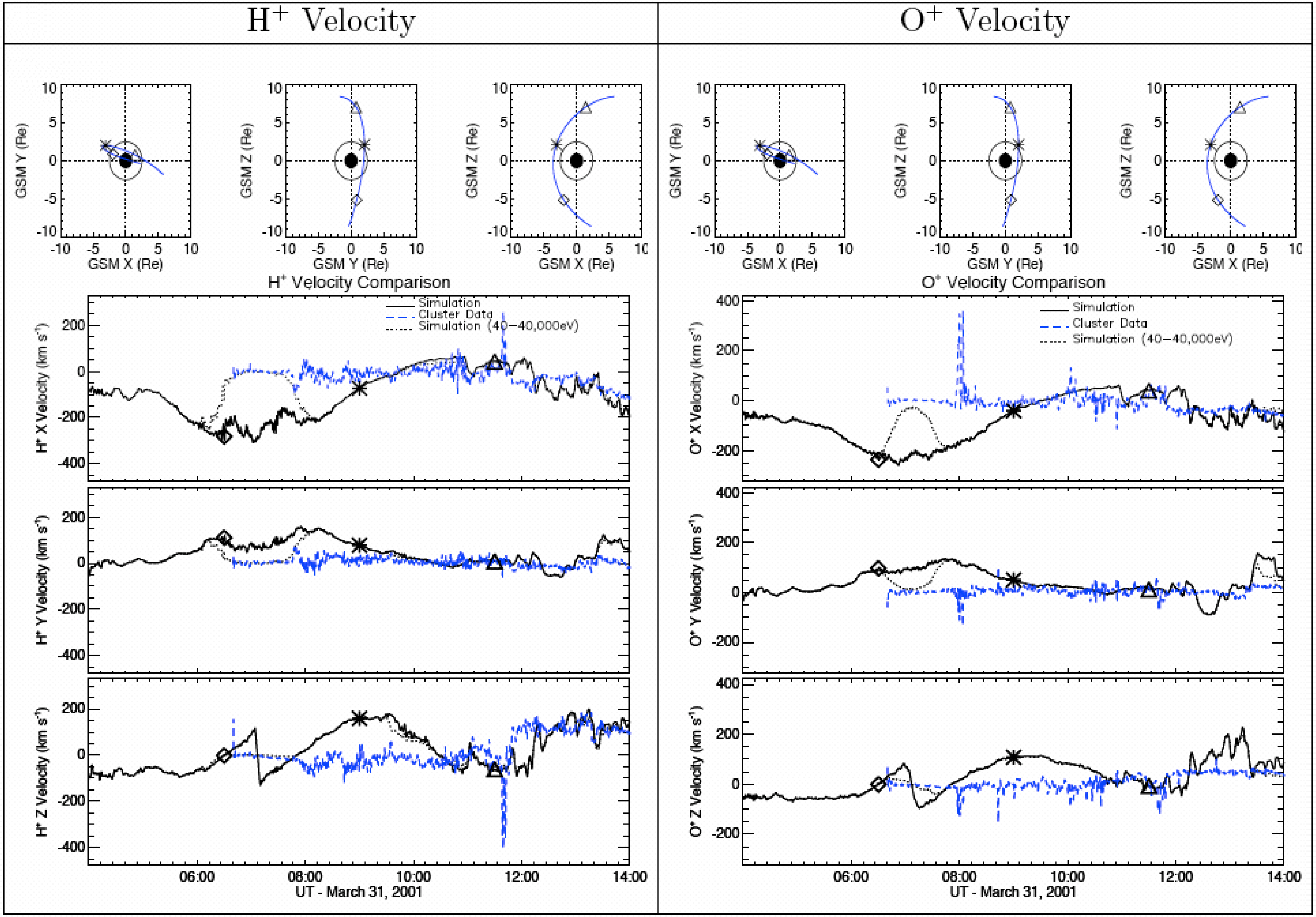


Magnetic Field vs Goes 8 Satellite

Multi-fluid MHD with O^+ outflow | Single-fluid MHD with no outflow

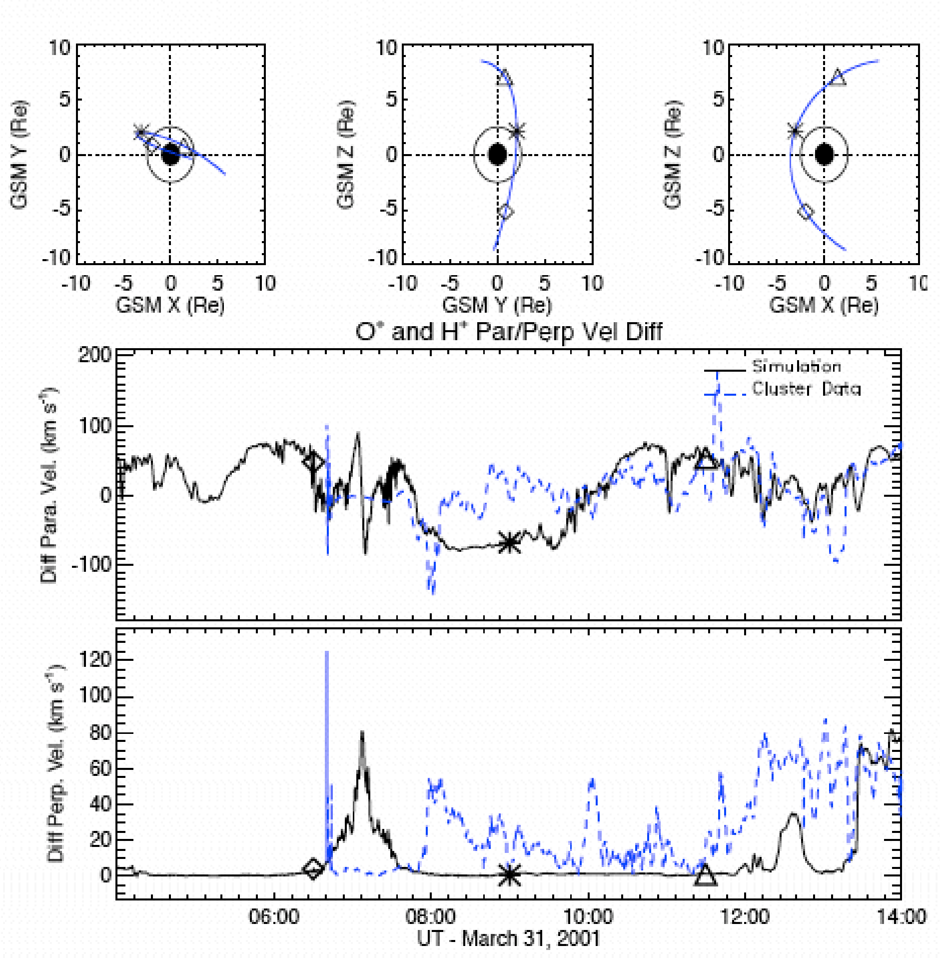


Velocities vs Cluster Satellite

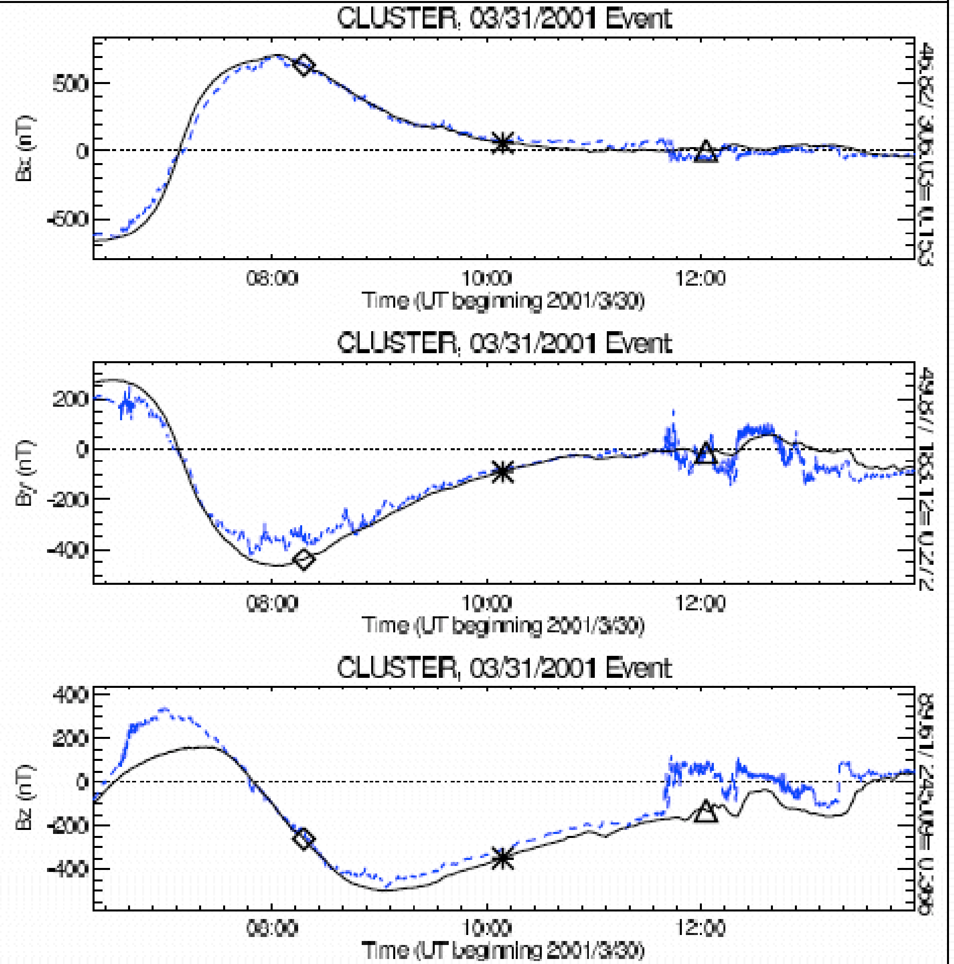


Velocity Differences and Magnetic Field

O^+ and H^+ Par/Perp Velocity Diff

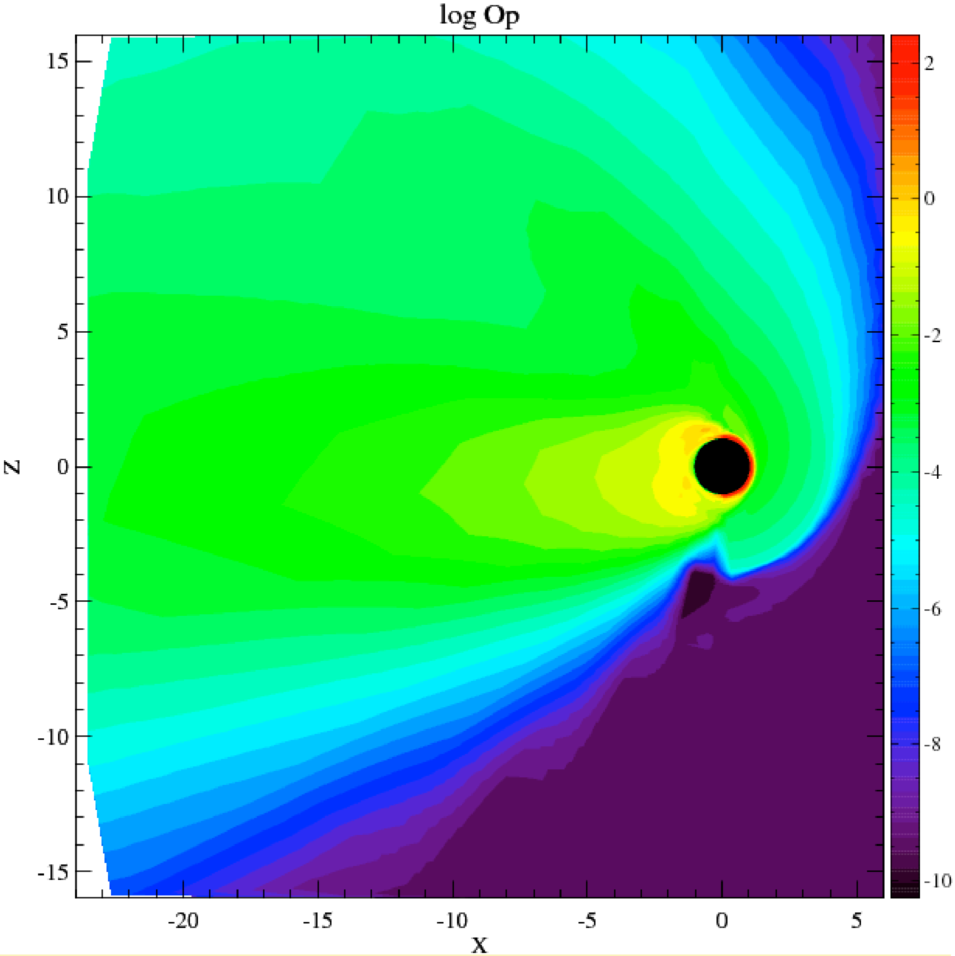


Magnetic Field

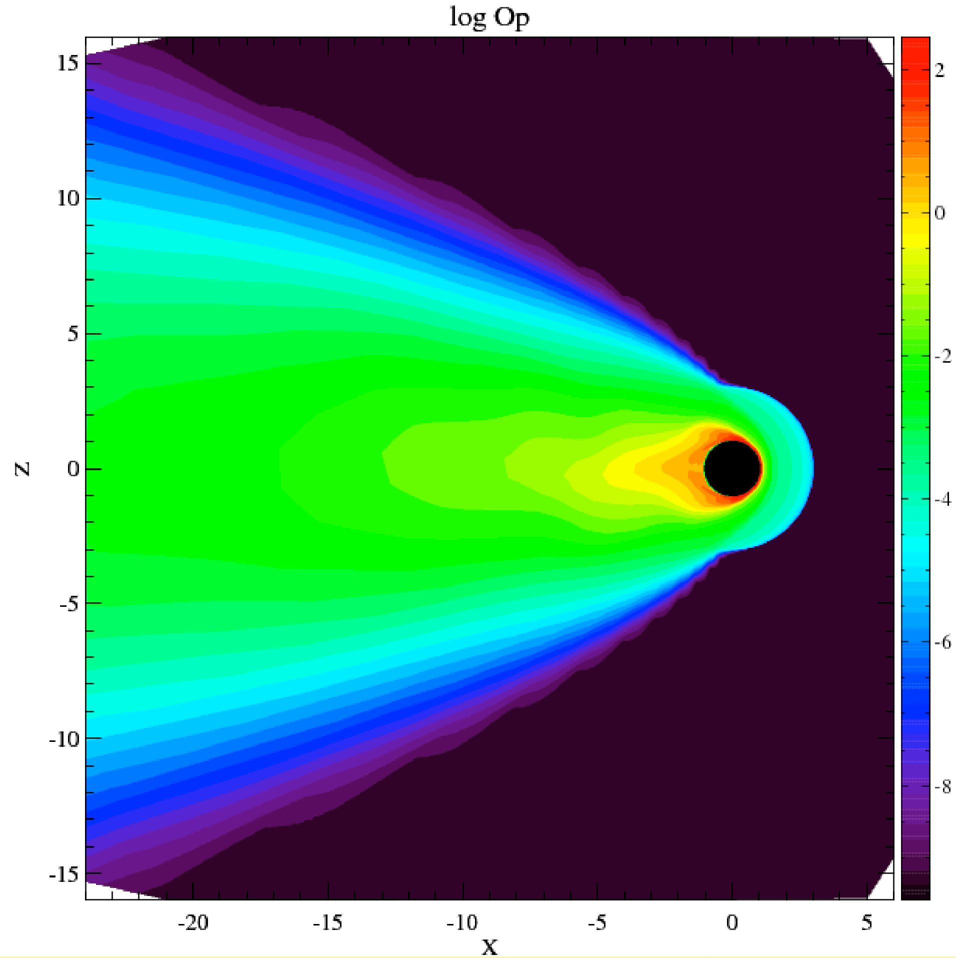


O^+ Escape from Mars Ionosphere

Multi-fluid MHD



Multi-species MHD



Summary

- **Added two-fluid (electron + ion) and anisotropic MHD to BATS-R-US.**
  - Initial verification tests pass
  - Preliminary magnetosphere runs look reasonable/interesting
  - Will be coupled with HEIDI, RAM, PWOM
  - Will be applied to reconnection, solar wind
- **Multi-fluid MHD is fairly well tested and working**
  - Coupled with RCM and PWOM
  - Mars ionosphere-solarwind interaction, in progress
  - Earth magnetosphere (Glocer et al, 2009, JGR)
  - Outer heliosphere (Opher et al, 2009, Nature)
- **New features are transferred to CCMC once they become robust**

Resistive Hall MHD with electrons and anisotropic ion pressure

Mass conservation: $$\frac{\partial \rho}{\partial t} + \nabla\cdot(\rho\mathbf{u}) = 0$$

Momentum: $$\frac{\partial \rho\mathbf{u}}{\partial t} + \nabla\cdot(\rho\mathbf{uu} + pI) = \mathbf{J}\times\mathbf{B}$$

$$\frac{\partial \rho\mathbf{u}}{\partial t} + \nabla\cdot(\rho\mathbf{uu} + P) = en_e(\mathbf{E} + \mathbf{u}\times\mathbf{B})$$

$$P = (p_\perp + p_e)I + (p_\parallel - p_\perp)\mathbf{bb} \qquad \mathbf{b} = \mathbf{B}/B$$

Induction: $$\frac{\partial \mathbf{B}}{\partial t} + \nabla\times\mathbf{E} = 0$$

Pressure: $$\frac{\partial p}{\partial t} + \nabla\cdot(p\mathbf{u}) = (\gamma-1)\left[-p\nabla\cdot\mathbf{u} + \eta\mathbf{J}^2 + \frac{3\eta e^2 n_e}{M_i}(p_e - p)\right]$$

$$\frac{\partial p_\perp}{\partial t} + \nabla\cdot(p_\perp\mathbf{u}) = +\frac{2\eta e^2 n_e}{M_i}(p - p_e) - p_\perp\nabla\cdot\mathbf{u} + p_\perp\mathbf{b}\cdot(\nabla\mathbf{u})\cdot\mathbf{b}$$

$$\frac{\partial p_\parallel}{\partial t} + \nabla\cdot(p_\parallel\mathbf{u}) = +\frac{2\eta e^2 n_e}{M_i}(p - p_e) - 2p_\parallel\mathbf{b}\cdot(\nabla\mathbf{u})\cdot\mathbf{b}$$

Electron pressure:

$$\frac{\partial p_e}{\partial t} + \nabla\cdot(p_e\mathbf{u}_e) = (\gamma-1)\left[(-p_e\nabla\cdot\mathbf{u}_e + \eta\mathbf{J}^2 + \frac{3\eta e^2 n_e}{M_i}(p - p_e) + \nabla\cdot(\kappa\mathbf{bb}\cdot\nabla T_e)\right]$$

Electron velocity: $$\mathbf{u}_e = \mathbf{u} - \frac{\mathbf{J}}{en_e}$$

Electric field: $$\mathbf{E} = -\mathbf{u}\times\mathbf{B} + \eta\mathbf{J} + \frac{\mathbf{J}\times\mathbf{B}}{en_e} - \frac{\nabla p_e}{en_e}$$

Current: $$\mathbf{J} = \nabla\times\mathbf{B}$$

Multi-Ion MHD Derived

Momentum equations for electrons with charge $-e$ and ion fluids s with charge q_s

$$\frac{\partial \rho_s \mathbf{u}_s}{\partial t} + \nabla \cdot (\rho_s \mathbf{u}_s \mathbf{u}_s + I p_s) = + n_s q_s (\mathbf{E} + \mathbf{u}_s \times \mathbf{B}) + S_{\rho_s \mathbf{u}_s}$$

$$\frac{\partial \rho_e \mathbf{u}_e}{\partial t} + \nabla \cdot (\rho_e \mathbf{u}_e \mathbf{u}_e + I p_e) = - n_e e (\mathbf{E} + \mathbf{u}_e \times \mathbf{B}) + S_{\rho_e \mathbf{u}_e}$$

Express electric field from electron momentum equation neglecting small terms:

$$\mathbf{E} = -\mathbf{u}_e \times \mathbf{B} - \frac{1}{e n_e} \nabla p_e + \eta \mathbf{J}$$

Obtain electron density from charge neutrality and electron velocity from current:

$$n_e = \frac{1}{e} \sum_s n_s q_s$$

$$\mathbf{u}_e = -\frac{\mathbf{J}}{e n_e} + \mathbf{u}_+$$ where the charged averaged ion velocity is $$\mathbf{u}_+ = \frac{\sum_s n_s q_s \mathbf{u}_s}{e n_e}$$

The electron pressure p_e is either a fixed fraction of total ion pressure, or we solve

$$\frac{\partial p_e}{\partial t} + \nabla \cdot (p_e \mathbf{u}_e) = -(\gamma - 1) p_e \nabla \cdot \mathbf{u}_e + S_{p_e}$$

Multi-Ion MHD Equations

For each ion fluid *s* (neglecting resistive terms):

$$\frac{\partial \rho_s}{\partial t} + \nabla \cdot (\rho_s \mathbf{u}_s) = S_{\rho_s}$$

$$\frac{\partial \rho_s \mathbf{u}_s}{\partial t} + \nabla \cdot (\rho_s \mathbf{u}_s \mathbf{u}_s + I p_s) = \frac{n_s q_s}{n_e e}(\mathbf{J} \times \mathbf{B} - \nabla p_e) + n_s q_s (\mathbf{u}_s - \mathbf{u}_+) \times \mathbf{B} + S_{\rho_s \mathbf{u}_s}$$

$$\frac{\partial p_s}{\partial t} + \nabla \cdot (p_s \mathbf{u}_s) = -(\gamma - 1) p_s \nabla \cdot \mathbf{u}_s + S_{p_s}$$

Induction equation (neglecting Hall term):

$$\frac{\partial \mathbf{B}}{\partial t} - \nabla \times (\mathbf{u}_+ \times \mathbf{B}) = 0$$

where the charge-averaged ion-velocity is $\mathbf{u}_+ = \frac{\sum_s n_s q_s \mathbf{u}_s}{e n_e}$

Two-Stream Instability

- **Perpendicular ion velocities are coupled through the magnetic field**
- **Parallel ion velocities are not coupled by the multi-ion MHD equations.**
- Two-stream instability restricts the velocity differences parallel to **B**
  - We cannot resolve the two-stream instability
  - Use a simple ad-hoc friction source term in the momentum equations:

$$S_{\rho \mathbf{u}_s}^{friction} = \frac{1}{\tau_c} \sum_{q \neq s} \min(\rho_s, \rho_q)(\mathbf{u}_q - \mathbf{u}_s) \left(\frac{|\mathbf{u}_s - \mathbf{u}_q|}{u_c} \right)^{\alpha_c}$$

  - Using the minimum of the two densities makes the friction uniformly effective in regions of low and high densities.
  - τ_C is the time scale, u_C is the cut-off velocity, α_C is the cut-off exponent
  - Currently we use fixed parameters.
  - We will explore physics based parameter setting and formulas in the future.

Multi-Ion MHD

For each ion fluid *s* (neglecting resistive terms):

$$\frac{\partial \rho_s}{\partial t} + \nabla \cdot (\rho_s \mathbf{u}_s) = S_{\rho_s}$$

$$\frac{\partial \rho_s \mathbf{u}_s}{\partial t} + \nabla \cdot (\rho_s \mathbf{u}_s \mathbf{u}_s + I p_s) = \frac{n_s q_s}{n_e e}(\mathbf{J} \times \mathbf{B} - \nabla p_e) + n_s q_s (\mathbf{u}_s - \mathbf{u}_+) \times \mathbf{B} + S_{\rho_s \mathbf{u}_s}$$

Cannot be written in conservative form

Gyration of ions around each other. Can be stiff.

$$\frac{\partial p_s}{\partial t} + \nabla \cdot (p_s \mathbf{u}_s) = -(\gamma - 1) p_s \nabla \cdot \mathbf{u}_s + S_{p_s}$$

We can also solve for *kinetic* energy density $e_s = \rho_s \mathbf{u}_s^2/2 + p_s/(\gamma - 1)$

$$\frac{\partial e_s}{\partial t} + \nabla \cdot [(e_s + p_s)\mathbf{u}_s] = \mathbf{u}_s \cdot \left[\frac{n_s q_s}{n_e e}(\mathbf{J} \times \mathbf{B} - \nabla p_e) + n_s q_s (\mathbf{u}_s - \mathbf{u}_+) \times \mathbf{B}\right] + S_{e_s}$$

Finally the induction equation with or without the Hall term becomes

$$\frac{\partial \mathbf{B}}{\partial t} - \nabla \times (\mathbf{u}_e \times \mathbf{B}) = 0 \quad \text{or} \quad \frac{\partial \mathbf{B}}{\partial t} - \nabla \times (\mathbf{u}_+ \times \mathbf{B}) = 0$$

Conservation

- **Multi-ion MHD equations cannot be written in conservation form.**
- **We would still like to maintain conservation for total ion fluid.**
- **Scheme:**
  - Solve for total ion fluid and individual ion and neutral fluids.
  - Use conservative scheme for total ion fluid, and non-conservative (there is no other choice) for the individual ion fluids.
  - Distribute total mass, pressure, and momentum (if all ions move the same direction) among the ion fluids proportionally to the individual solutions, e.g.:

$$p_s^{n+1} = p_s^* \frac{p^{n+1}}{\sum_q p_q^*}$$

  - For ion momentum components with mixed signs we do the opposite:

$$(\rho \mathbf{u})^{n+1} = \sum_s (\rho_s \mathbf{u}_s)^*$$

Positivity

- **Positivity is difficult to maintain in empty regions where some of the fluids do not occur.**
- **In some problems we can identify effectively single-ion regions based on geometry and/or physical state.**
  - For example the solar wind has high Mach number.
- **In other problems we have to check after every time step if any of the fluids have very small density or pressure relative to the total.**
- **For *minor* fluids**
  - Density is set to a small fraction ($\sim 10^{-4}$) of the total ion density.
  - Velocity and temperature are set to the same as for the total ion fluid.
  - This is a physically meaningful state that can interact properly with the truly multifluid regions.

Stability

- **Equations can either be solved with the fully implicit scheme.**
- **Or we can use an explicit scheme with point-implicit source terms:**

$$(\rho_s \mathbf{u}_s)^{n+1} = (\rho_s \mathbf{u}_s)^n - \Delta t \nabla \cdot \mathbf{F}^n + \Delta t S_{\rho \mathbf{u}_s}^{n+1}$$

$$+\Delta t \left[\frac{q_s}{M_s} (\rho_s \mathbf{u}_s - \rho_s \mathbf{u}_+)^{n+1} \times \mathbf{B}^n + \frac{n_s^n q_s}{n_e^n e} (\mathbf{J}^n \times \mathbf{B}^n - \nabla p_e^n) \right]$$

where M_S is the mass of ion *s*.

- The linear equations can be solved in every grid cell independently.
- The unknowns are the momenta of the ion fluids.
- The three spatial components are coupled by the artificial friction term.
- We use an analytic Jacobian matrix for sake of efficiency and accuracy.