Modeling of geomagnetically induced current flow in high-voltage power transmission systems

Antti Pukkinen

June, 2008

1 Estimating geomagnetically induced current from the time derivative of the magnetic field

In this section, a simple method that one can use to estimate geomagnetically induced current (GIC) at individual high-voltage power transmission system nodes is derived. First, basic mathematical relations associated with the GIC phenomenon are reviewed and the expressions used in the method are derived. Then, the method is "tailored" for application with an individual node of the North American power transmission system.

1.1 Some basic mathematics

Although in detailed investigations more complex mathematical relations may need to be utilized, GIC is to a good approximation linear function of the local geoelectric field, i.e.

$$GIC(t) = aE_x(t) + bE_y(t) \tag{1}$$

where t denotes time, E_x and E_y are the horizontal components of the geoelectric field and a and b are the so-called system parameters that depend on the electrical resistances and the topology of the power transmission system under investigation. Following the standard convention used for the geomagnetic field recordings, x-axis points toward geographical north and y-axis toward geographical east. Note that Eq. (1) is basically just Ohm's law.

The horizontal geoelectric field in Eq. (1) can be obtained in many situations to a good approximation by computing

$$E_x(t) = \frac{1}{\sqrt{\pi \mu_o \sigma}} \int_{-\infty}^{t} \frac{dY(t')/dt'}{\sqrt{t-t'}} dt' \tag{2}$$

$$E_y(t) = -\frac{1}{\sqrt{\pi \mu_o \sigma}} \int_{-\infty}^{t} \frac{dX(t')/dt'}{\sqrt{t-t'}} dt' \tag{3}$$

where dY/dt' and dX/dt' are the time derivatives of the horizontal components of the magnetic field, μ_0 is the vacuum permeability and σ is the ground conductivity. To arrive at Eqs. (2) and (3), one needs to assume that the magnetic field on the ground varies as a linear function of the horizontal distance, i.e. the second spatial derivative of the field vanishes, and that the ground is homogeneous.

From Eqs. (2) and (3) it is clear that the time derivative of the magnetic field can be considered as a "driver" of the geoelectric field and consequently via Eq. (1) also the driver of GIC. As can be seen from the denominators of Eqs. (2) and (3), the geoelectric field at a given time instant t contains information about the past values of the time derivative of the magnetic field. However, the greatest weight is on the most recent magnetic field values. Thus, to simplify the GIC modeling process further, one can try making an approximation

$$E_x(t) \approx \alpha \cdot dY(t)/dt \tag{4}$$

$$E_y(t) \approx \beta \cdot dX(t)/dt \tag{5}$$

where α and β are some constants. By inserting Eqs. (4) and (5) into Eq. (1) one finally obains

$$GIC(t) \approx a\alpha \cdot dY(t)/dt + b\beta \cdot dX(t)/dt \tag{6}$$

which gives GIC as a linear function of the time derivatives of the horizontal components of the magnetic field. Clearly, Eq. (6) is a result of some violent approximations and should be used only to roughly estimate GIC. See *Pulkkinen* (2003) and Section 2 for further details on the GIC modeling process.

1.2 Estimating GIC levels at the North American high-voltage power transmission system

To test the adequacy of the approximations used to arrive at Eq. (6), GIC data from one of the North American high-voltage power transmission system nodes and magnetic field data from nearby Ottawa geomagnetic observatory were used (Fig. 1). Fig. 2 shows one-minute GIC and the magnetic field data from these locations for the period of October 24 - November 1, 2003, which contains the Halloween storm event of October 29-31, 2003. The time derivatives of the magnetic field were evaluated by using the three-point formula

$$f'(t) = \frac{f(t - \Delta t) - f(t + \Delta t)}{2\Delta t} \tag{7}$$

Fig. 3 shows the scatter plots of the data in Fig. 2. It can be seen that while dY/dt does not show any apparent correlation with GIC, dX/dt does show a weak linear relationship with GIC. In another words, it is reasonable to set $a = 0$ in Eq. (6) and model GIC by using a relation

$$GIC(t) \approx k \cdot dX(t)/dt \tag{8}$$

where the linear coefficient k can be obtained by a linear fit to the data in the top panel of Fig. 3. Least-squares fit gives $k = -8.6$ A·s/nT and as is seen from Fig. 3, the linear fit does represent the general trend in the data. Consequently, the model given by Eq. (8) can be used to estimate GIC at the node based on the known time derivative of the x-component of the magnetic field observed at the vicinity of the GIC site. It should, however, be noted that the generated simple model is valid only for the chosen particular GIC site. Generalizations to other sites require case-by-case validation of the simplified modeling process.

2 Full simulation of the geomagnetically induced current flow in a high-voltage power transmission system

In this section, the basic steps required for the fully first-principles modeling of GIC in an arbitrary power transmission system are briefly reviewed.

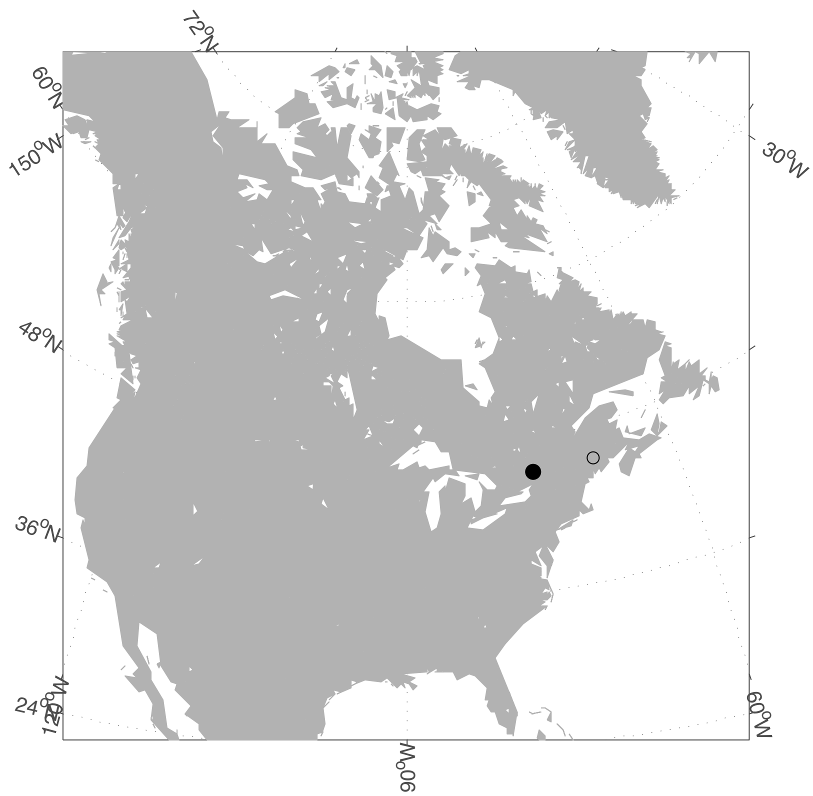

Figure 1: The locations of the GIC station (circle, 45^{o}N, 69^{o}W) and the Ottawa geomagnetic observatory (dot, 45.4^{o}N, 75.6^{o}W) used in the analysis. Geographic coordinates are used.

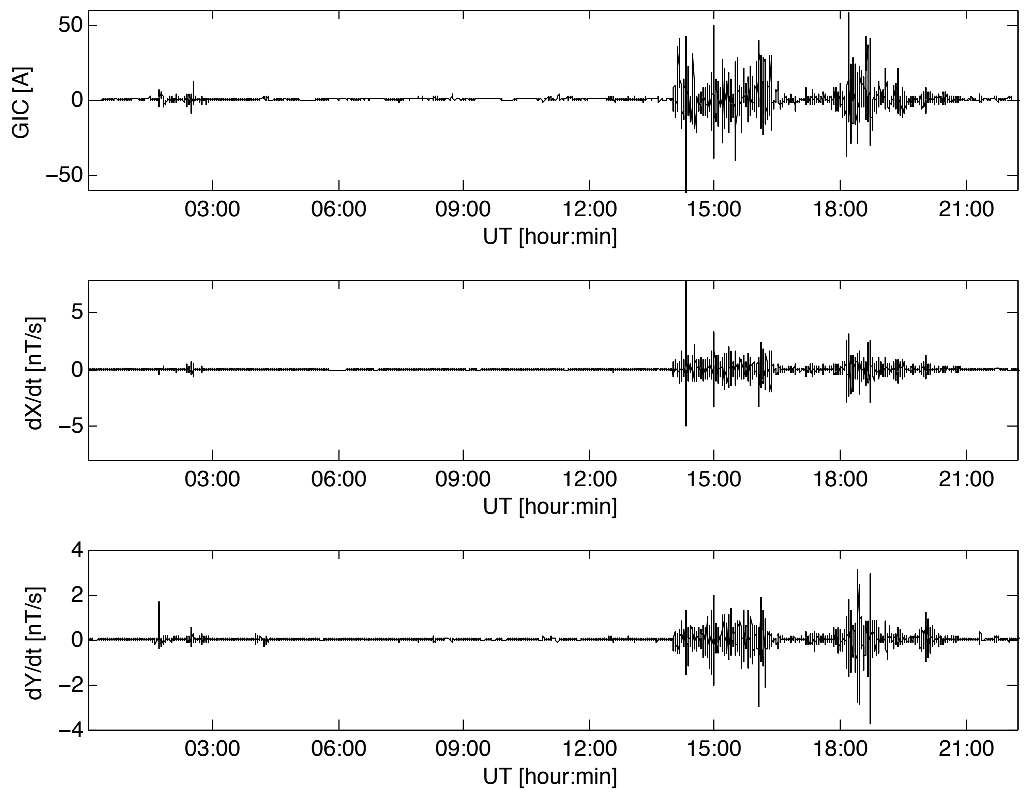

Figure 2: One-minute GIC and magnetic field data from the stations shown in Fig. 1. The data is shown for the period of October 24 - November 1, 2003.

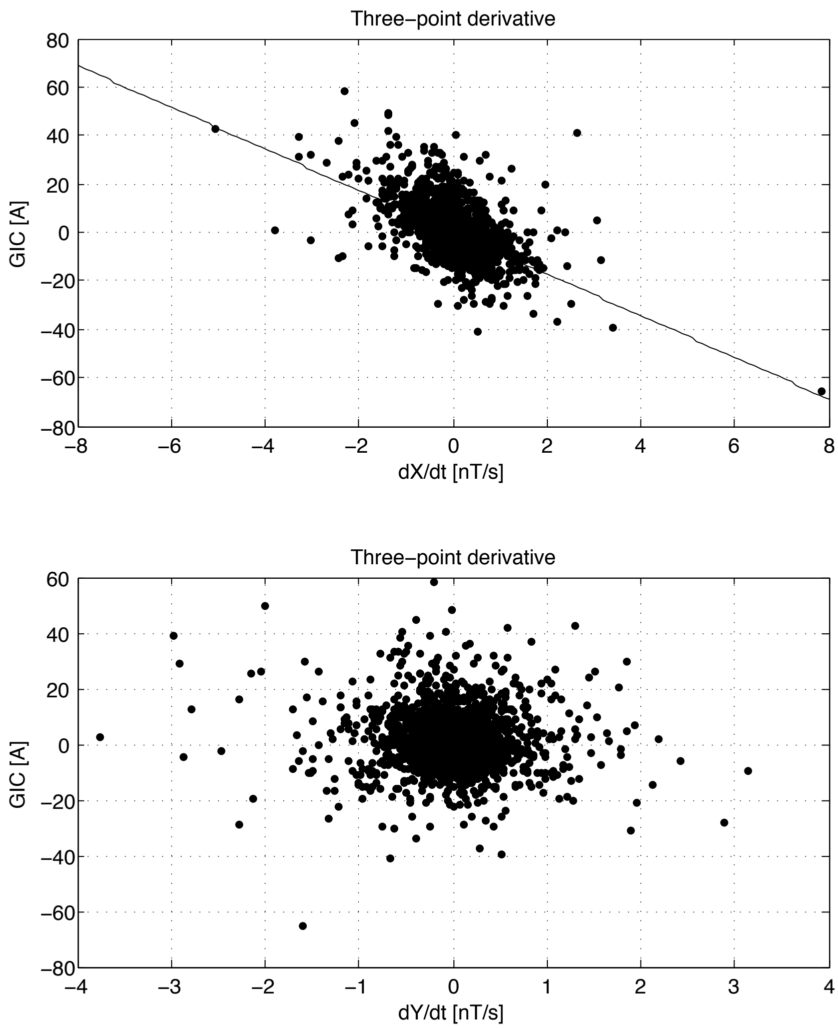

Figure 3: Scatter plots of the data in Fig. 2. The line in the panel on the top shows linear fit to the data. The fit gives a relation $GIC(t) \approx -8.6 \cdot dX(t)/dt$.

2.1 The derivation of the ground conductivity model and the system parameters

As was seen in the section above, the first two steps in any detailed GIC modeling involves the determination of the ground conductivity structure and the system parameters. The system parameters can be computed for any point in the power transmission system if the electrical resistances and the topology of the system are known. By using Ohm's and Kirchoff's laws GIC flowing through the grounding points of a power grid can be obtained from the matrix equation (*Lehtinen and Pirjola*, 1985)

$$\mathbf{I}^e = (\mathbf{1} + \mathbf{Y}\mathbf{Z}^e)^{-1}\mathbf{J}^e \tag{9}$$

where $\mathbf{Z}^e$ is the earthing impedance matrix and

$$J_i^e = -\sum_{j \neq i} \frac{V_{ij}}{R_{ij}} \tag{10}$$

and $\mathbf{Y}$ is the network admittance matrix

$$Y_{ij} = \begin{cases} -\frac{1}{R_{ij}} & i \neq j \\ \sum_{k \neq i} \frac{1}{R_{ik}} & i = j \end{cases} \tag{11}$$

and

$$-V_{ij} = \int_j^i \mathbf{E} \cdot d\mathbf{s} \tag{12}$$

where $\mathbf{E}$ is the horizontal geoelectric field driving GIC, indices i and j indicate grounded nodes of the system and R_{ij} is the line resistance between nodes i and j. The integration between nodes i and j is made along the conductor connecting the nodes. Alternatively, if both the geomagnetic field and GIC observations are available, one can determine the system parameters by the cross-correlation technique introduced by Pulkkinen et al. (2007b). The technique adjusts the parameters a and b in Eq. (1) by studying the correlations between GIC and the time derivative of the magnetic field.

Although it is often justified to assume that ground is homogeneous, in detailed GIC studies one should use more complex ground conductivity structures. The local conductivity structure can be estimated, in principle, from the known geological properties of the crust. However, if again both the geomagnetic field and GIC observations are available, one can invert the

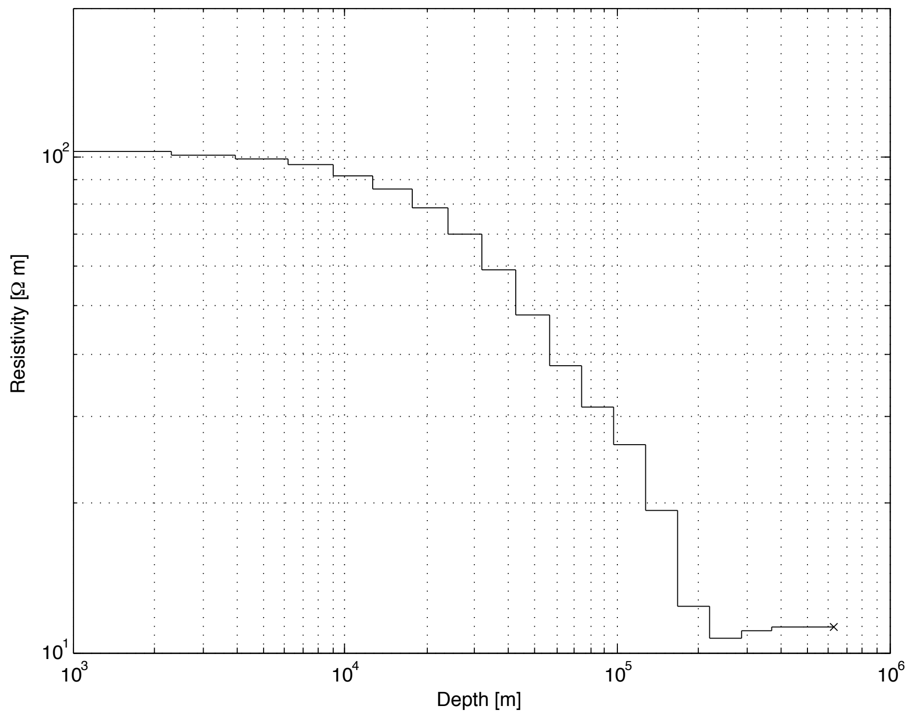

Figure 4: One-dimensional ground conductivity model derived by using the geomagnetic field and GIC observations carried out at the stations shown in Fig. 1. The cross indicates the resistivity (inverse of the conductivity) of the terminating half-space.

conductivity structure by using the techniques familiar from the magnetotelluric sounding of the Earth's crust and upper mantle (approximately within depths of 1000 km $> |z| >$ 100 m) (Pulkkinen et al., 2007b). The determination of the layered ground structure is quite involved process and requires a robust determination of the surface impedance mapping geomagnetic field fluctuations to geoelectric field fluctuations and the the usage of the surface impedance in ill-posed non-linear inversion resulting in an one-dimensional ground conductivity structure (see Fig. 4).

2.2 Computation of the geoelectric field and GIC

If in addition to the system parameters and the local ground conductivity structure also the spatiotemporal behavior of the ionospheric current fluctuations are known, one can carry out fully first-principles based modeling of GIC. First, by using the known ground conductivity structure and the spatiotemporal behavior of the ionospheric currents, one computes the geoelectric field. A computationally efficient and a flexible way to do this is to apply the so-called Complex Image Method (CIM) (Pirjola and Viljanen, 1998). In CIM the electromagnetic field induced on the surface of the Earth is represented by image currents placed within the Earth. The complex depth (this is the reason for the name of the method) of the image currents depends on the frequency of the ionospheric current fluctuations and on the ground conductivity structure. The only significant drawback of CIM is that the method is applicable only to situations where the field-aligned currents flowing to and from the ionosphere are perpendicular to the ionospheric plane. This restricts the usage of the method to only high-latitude situations.

Once the geoelectric field $\mathbf{E}$ is known, one can use Eqs. (9)-(12) to compute the GIC in the system, or coefficients a and b in Eq. (1) determined by means of the cross-correlation method to map the geoelectric field to GIC. The first experimental fully first-principles-based real-time GIC forecasting system established at Community Coordinated Modeling Center (CCMC) uses the cross-correlation method and the inversion of the geomagnetic field and GIC observations to derive the system parameters and the ground conductivity structure. Then, CIM is used to compute the geoelectric field from the ionospheric output of a global MHD simulation of the Earth's magnetosphere-ionosphere system (Pulkkinen et al., 2007a). CCMC's real-time forcasting system is being developed in collaboration with the Electric Power Research Institute.

References

Lehtinen, M., and R. Pirjola, Currents Produced in Earthed Conductor Networks by Geomagnetically-Induced Electric Fields, *Annales Geophysicae*, Vol. 3, No. 4, 479-484, 1985.

Pirjola, R., and Viljanen, A., Complex image method for calculating electric and magnetic fields produced by and auroral electrojet if finite length, *Annales Geophysicae*, 16, 1434-1444, 1998.

Pulkkinen, A., Geomagnetic Induction During Highly Disturbed Space Weather Conditions: Studies of Ground Effects, *Finnish Meteorological Institute, Contributions*, 42, 2003. (E-thesis available at http://ethesis.helsinki.fi/julkaisut/mat/fysik/vk/pulkkinen/)

Pulkkinen, A., M. Hesse, M. Kuznetsova and Lutz Rastätter, First-principles modeling of geomagnetically induced electromagnetic fields and currents from upstream solar wind to the surface of the Earth, *Annales Geophysicae*, 25, 881-893, 2007a.

Pulkkinen, A., R. Pirjola, and A. Viljanen, Determination of ground conductivity and system parameters for optimal modeling of geomagnetically induced current flow in technological systems, *Earth, Planets and Space*, 99, 999-1006, 2007b.