Turbulence and Complexity throughout the Heliosphere

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Introduction

THE "INCONVENIENT TRUTH": multiscale stochastic space environment



	Solar Wind	Magnetosheath	Magnetotail
Re	1×10^{14}	1×10^{12}	5×10^{12}
Re_m	3×10^{14}	1×10^{13}	1×10^{13}

J Borovsky, H. Funsten JGR 2003







Multiscaling in space plasmas



Why should we care ?



Sun

Structure of the Sun and the coronal heating problem



Examples of non-potential coronal structures

TRACE loop arcade









Neutral line + coronal hole

Differential rotation

Solar and Heliospheric Observatory (SOHO) and Solar TErrestrial RElations Observatory (STEREO) spacecraft





Multiscale intermittency in the solar corona : SOC or turbulence?



Uritsky, V.M., M. Paczuski, D. Davila, and S. I. Jones, PRL, 99(2), Art. No. 025001, 2007.

Spatiotemporal tracking of multiple active regions in SOHO EIT images (195 A)



Lifetime probability distributions of coronal active regions



Probability distributions of coronal active regions over integrated luminosity



Spatial scaling of high-order structure functions of SOHO EIT images



Full-disk SOHO MDI magnetogram (left) and STEREO EUVI image (right) showing the studied quiet solar region.



Spatiotemporal plots showing about 1/4 of the photospheric (MDI) and coronal (EUVI) events detected at respectively p = 99% and 95% percentile levels.



Comparison of probability distributions of photospheric and coronal events obtained for two combinations of thresholds yielding comparable numbers of detected events in each data sets.

p	MDI threshold	EUVI threshold
95.0~%	$13.5 \ (n = 38242)$	$202 \ (n = 4124)$
97.0~%	$17.3 \ (n = 19114)$	$217 \ (n = 3005)$
99.0~%	$32.3 \ (n = 5912)$	$258 \ (n = 1269)$
99.5~%	$42.0 \ (n = 3410)$	292 $(n = 686)$



Energy budget (photosphere)

Energy dissipation (corona)







Understanding multiscale topology of coronal magnetic network







Earth's magnetosphere

Happy dreams...



... and harsh reality



Multiscale organization of geomagnetic activity







Hierarchy of magnetospheric

disturbances

- Geomagnetic storms
- Full-size substorms
- Substorm activations
- Pseudo-breakups
- Bursty bulk flows (BBFs)
- Impulsive structure of BBF events
- Plasma turbulence

Earth's magnetosphere as an open nonlinear dissipative system





UVI filter	LBH-L (N ₂) λ =160-180 nm
Integration time	36.8 s
Spatial resolution	70×70 km
Temporal resolution	184 c, 37 c
Satellite altitudes	> 6 Re
MLT range MLat range	$2000 - 0400 \\ 55^{\circ} - 90^{\circ}$
Observation periods	Jan-Feb 1997 Jan-Feb 1998
Number of processed images	~30 000

Detection and spatiotemporal tracking of active emission regions on POLAR UVI images



UVI 981205 11:02:24 UT LBHL

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28

Lifetime probability distributions of auroral emission regions for different time periods and resolution



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Probability distributions of emission regions of the spatiotemporal avalanche size and total energy output



Power-law statistics of auroral emission events (Uritsky et al., JGR 2002; Kozelov et al., GRL 2004)



POLAR: $T = 10^2 - 10^4 s$, $A = 10^4 - 10^7 km^2$ TV: $T = 10^0 - 10^2 s$, $A = 10^1 - 10^3 km^2$





Modeling

Fractal models: the geometry of scaleinvariance



"Smooth" treatment of multiscale systems

$$\begin{split} & \underbrace{\frac{\partial\rho}{\partial t} + \nabla \cdot \rho \vec{u} = 0,} \\ & \underbrace{\frac{\partial(\rho \vec{u})}{\partial t} + \nabla \cdot (\rho \vec{u} \vec{u}) = -\nabla p + \vec{j} \times \vec{B} + \nabla \cdot \sigma,} \\ & \frac{\partial(\rho \vec{u})}{\partial t} + \nabla \cdot (\rho \vec{u} \vec{u}) = -\nabla p + \vec{j} \times \vec{B} + \nabla \cdot \sigma, \\ & \underbrace{\frac{\partial\rho}{\partial t} + \vec{u} \cdot \nabla p + \gamma p \nabla \cdot \vec{u} = Q,} \\ & \underbrace{\frac{\partial\vec{B}}{\partial t} = -\nabla \times \vec{E} = \nabla \times (\vec{u} \times \vec{B}) + \eta \nabla^2 \vec{B}.} \end{split}$$

"Smooth" treatment of multiscale systems

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$$\begin{split} &\frac{\partial\rho}{\partial t} + \nabla \cdot \rho \vec{u} = 0, \\ &\frac{\partial(\rho \vec{u})}{\partial t} + \nabla \cdot (\rho \vec{u} \vec{u}) = -\nabla p + \vec{j} \times \vec{B} + \nabla \cdot \vec{u} \\ &\frac{\partial p}{\partial t} + \vec{u} \cdot \nabla p + \gamma p \nabla \cdot \vec{u} = Q, \\ &\frac{\partial \vec{B}}{\partial t} = -\nabla \times \vec{E} = \nabla \times (\vec{u} \times \vec{B}) + \eta \nabla^2 \vec{B}. \end{split}$$



Two paradigms for understanding multiscale complexity

Turbulence

Complexity emerges as a result of fluid instabilities which typically propagate from large to small (dissipative) scales. The intermediate (inertial) range exhibits scale-free correlations in space and in time

Self-organized criticality

Complexity results from cascades of discrete "toppling" events creating inverse cascades of energy transport called avalanches. The statistics of these events are scale-free, but the background can be uncorrelated.





Phenomenology of the turbulent spectrum

HOMOGENEOUS FLUID CASCADE (Kolmogorov, 1941)



 $k_{in} \ll k \ll k_d$

$$E_k = \int d\Omega_k \hat{E}_k, \ E = \int_{\infty}^0 dk E_k$$

$$\epsilon_{in} = \epsilon_t = \epsilon_d := \epsilon$$

$$l_0 > l_1 > \dots > l_N, \ k_0 < k_1 < \dots < k_N$$

 $l_n = k_n^{-1} = 2^{-n} l_0, \ l_0 \sim L$

$$\tau_n \sim l_n / \delta v_n, \quad \delta v_n \equiv \delta v_{l_n}$$
$$E_n / \tau_n \sim \delta v_n^3 / l_n \sim \epsilon \quad \Rightarrow \quad \delta v_n \sim \epsilon^{1/3} l_n^{1/3}$$
$$\delta v_n^2 \simeq E_n \quad \simeq \int_{k_n}^{k_{n+1}} E_k dk \quad \simeq E_{k_n} k_n$$

$E_k \sim \epsilon^{2/3} k^{-5/3}$

INTERMITTENCY



3D incompressible MHD (Uritsky et al., *PRE*, 2010)

Phenomenology of the turbulent spectrum

HOMOGENEOUS FLUID CASCADE (Kolmogorov, 1941)



 $k_{in} \ll k \ll k_d$ $E_k = \int d\Omega_k \hat{E}_k, \quad E = \int_{\infty}^0 dk E_k$ $\epsilon_{in} = \epsilon_t = \epsilon_d := \epsilon$ $l_0 > l_1 > \dots > l_N, \quad k_0 < k_1 < \dots < k_N$ $l_n = k_n^{-1} = 2^{-n} l_0, \quad l_0 \sim L$ $\tau_n \sim l_n / \delta v_n, \quad \delta v_n \equiv \delta v_{l_n}$ $E_n / \tau_n \sim \delta v_n^3 / l_n \sim \epsilon \quad \Rightarrow \quad \delta v_n \sim \epsilon^{1/3} l_n^{1/3}$ $\delta v_n^2 \simeq E_n \quad \simeq \int_{k_n}^{k_{n+1}} E_k dk \quad \simeq E_{k_n} k_n$ $E_k \sim \epsilon^{2/3} k^{-5/3}$

PLASMA EFFECTS AT SMALL SCALES



Bale et al., *PRL* 2005; Schekochihin et al., *Plasma Phys. Control. Fusion* 2007

Examples of plasma turbulence near Earth





Scaling and avalanches in SOC models



$$|Z(\mathbf{i}) \rightarrow Z(\mathbf{i}) + \delta Z|$$

$$\mathbf{i}$$
)> $Z_c \Rightarrow \begin{cases} Z(\mathbf{i}) \rightarrow Z(\mathbf{i}) - \sum_{\mathbf{e}} \Delta Z(\mathbf{e}), \\ Z(\mathbf{i} + \mathbf{e}) \rightarrow Z(\mathbf{i} + \mathbf{e}) + \Delta Z(\mathbf{e}) \end{cases}$

$$\Lambda = \bigcup_{t=t_0}^{t_0+T} (t, \lambda_t), \quad \lambda_t = \left\{ \mathbf{i} \left| Z_t(\mathbf{i}) \ge Z_c \right\} \right\}$$
$$S = \sum_{t=t_0}^{t_0+T} \sum_{\mathbf{i}} \Theta \left(Z_t(\mathbf{i}) - Z_c \right)$$
$$E = \sum_{t=t_0}^{t_0+T} \sum_{\mathbf{i}} \left[\Theta \left(Z_t(\mathbf{i}) - Z_c \right) \times \sum_{\mathbf{e}} \Delta Z_t(\mathbf{e}) \right]$$



Criticality, renormalization group, and the universality of multiscale complexity



Power-law scaling at criticality

$$\begin{array}{c} R \to R^* \\ S \to S^* \end{array}$$

$$F(L) = a \times F(L \times b), \quad L = \{ \Delta \mathbf{x}, \Delta t, \Delta R, ... \}$$
$$b = \mathbf{1} + \delta, \ a = \mathbf{1} + \alpha \delta, \ \delta \ll \mathbf{1}$$

$$F(L+\delta L) - F(L) = -\alpha \delta F(L+\delta L)$$
$$\frac{F(L+\delta L) - F(L)}{\delta L} = -\alpha \frac{F(L+\delta L)}{L}$$

$$\frac{dF}{dL} = -\alpha \frac{F}{L} \longrightarrow F(L) \sim L^{-\alpha}$$
Scale-free: $\alpha \neq f(L)$

Sethna et al., Nature 410, 242-250, 2001

Measuring SOC: scaling exponents and relations

Avalanche
critical
exponents
$$p(S) = S^{-\tau_{s}} f_{s}(S/S_{c}), S_{c} \sim L^{D}$$

 $p(E) = E^{-\tau_{s}} f_{s}(E/E_{c}), E_{c} \sim L^{D_{s}}$
 $p(T) = T^{-\tau_{T}} f_{T}(T/T_{c}), T_{c} \sim L^{z}$ Scaling relationsDynamical
critical
exponents $\sigma(t) = \frac{1}{N} \sum_{k=1}^{N} \left[\int_{A_{k}(t+t_{0k})} dV \right], P_{s}(t) = n(t)/N$
 $\sigma \sim t^{\eta}; P_{s} \sim t^{-\delta}; \overline{S} = \int_{s_{1}}^{s_{2}} p(S|T) dS \sim T^{\kappa}$ $\eta + \delta + 1 = \kappa$ Fractal
critical
exponents $C(r) = \left\langle \left[\left\langle \left(w(r, t) - w(r', t) \right)^{2} \right\rangle_{r} \right]^{1/2} \right\rangle_{r}, r \equiv |r - r'|$
 $C(\tau) = \left\langle \left[\left\langle \left(w(r, t) - w(r, t') \right)^{2} \right\rangle_{\tau} \right]^{1/2} \right\rangle_{r}, \tau \equiv |t - t'|$
 $D_{p} = d - \alpha, D_{1} = 2 - \beta, N(r) - (1/r)^{D_{s}}$ $D = \alpha + D_{s}$

Toward realistic description of plasma turbulence









MODELING: 2-D resistive MHD model of the current sheet avalanching dynamics (A. Klimas, JGR 2004)



Spatiotemporal evolution of model parameters



Probability distributions of anomalous resistivity regions over dissipated energy (left) and lifetime (right)



Daughton et al., Nature Phys 2011





- What role does multiscale turbulence play?
- What is its fundamental physical mechanism?
- Does turbulence facilitate or inhibit fast Rx?
- Is energy conversion cominated by small or large scales?
- How to measure such Rx events?



Unresolved problems & future tasks

Is physics indeed scale free ?



"Mainstream" community



Case-by-case analysis

"Complexity" community



Ensemble averaging

Functional complexity of plasma behavior: the diversity of mechanisms



Two approaches to exploring complex dynamical systems



THANK YOU !