Visualizing Magnetic Field Topology in the Magnetotail using TRISTAN code

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Abstract

We discuss our work using critical point analysis to generate representation of the three-dimensional magnetic vector field topology of numerical magnetic vector data sets from TRISTAN code. Critical points are magnetic nulls, and located and characterized in a three-dimensional domain. The magnetic vector field curves and surfaces are integrated out along the principal directions of certain classes of critical points including the earth dipole magnetic field. Both the curves and surfaces are the characteristic ones. All the points, curves, and surfaces are uniquely linked to form a skeleton representing the tree-dimensional vector field topology.

When generated from the magnetic vector field near the magnetophere in a three-dimensional magnetic field, the skeleton includes the critical points, curves, and surfaces that provide a basis for understanding the three–dimensional topological structure of the reconnection. Two critical points on the earth magnetosphere that forms the three dimensional X points and the two connecting characteristic curves which do not have the third eigenvector components are used to investigate the change of the topological structure in magnetic reconnection.

1 Introduction

When computer graphics is introduced to a field of study, the visualization techniques to emerge firsts are the ones that most closely resemble the "pictures" already in use a familiar to those in the field. To the researcher, who having seen thousands of them, has learned to interpret them, such images may be more useful than a new representation that actually contains more information. However, direct visualization methods in which thousands of points, vectors or curves are displayed are inadequate for visualizing many complex data sets, and manually choosing a smaller set of elements for direct display is usually both time consuming and error prone.

The importance of topology in understanding magnetospheric dynamics combined with the difficulty of extracting topological magnetic field information with existing tools has motivated our efforts. This paper describes some of methods we have developed to automate the analysis and display of magnetic vector field topology in the near earth, especially, magnetotail regions that considered to be one of the most important part analyzing the space physics. We firsts discuss the basis of critical point analysis and classifications, and then discuss the algorithm of our visualization.

Topological concepts are very powerful because given the critical points in a magnetic vector field and the magnetic field curves or surfaces connecting them, one can infer the shape of other magnetic field curves or surfaces that are the topologically equivalent ones and hence to some extent the structure of the entire magnetic vector field.

2 Critical Points

Critical points or magnetic nulls are those points at which the magnitude of the magnetic field vector vanished. These points may be characterized according to the behavior of nearby magnetic field curves or surfaces. The set of the curves or surfaces which end on critical points are of special interests because they define the behavior of the magnetic vector field in the neighborhood of the point. If all the eigenvalues of critical points in the region we consider are hyperbolic, then the magnetic vector field topologies are uniquely determined only by the critical points, the magnetic field curves, and surfaces that are originated from the critical points [2]. The hyperbolic conditions are the case we are considering and this is the usual magnetic vector field configuration. Thus these particular sets of the critical points, curves, and surfaces can be used to define a skeleton that uniquely characterizes the magnetic vector field we are going to visualize.

For simplicity, we use $\mathbf{v} = (u, v, w)^t$ instead of $\mathbf{B} = (B_x, B_y, B_z)^t$ as the magnetic vector field. We first Taylor expand the magnetic vector field $\mathbf{v}(x, y, z)^t$ around the critical point $\mathbf{x} = (x_0.y_0, z_0)^t$ to the first order:

$$\begin{aligned}\mathbf{v} &= \frac{d\mathbf{x}}{dt} \\ &= \begin{pmatrix} u(x_0, y_0, z_0) \\ v(x_0, y_0, z_0) \\ w(x_0, y_0, z_0) \end{pmatrix} + \begin{pmatrix} \frac{\partial u}{\partial x} & \frac{\partial u}{\partial y} & \frac{\partial u}{\partial z} \\ \frac{\partial v}{\partial x} & \frac{\partial v}{\partial y} & \frac{\partial v}{\partial z} \\ \frac{\partial w}{\partial x} & \frac{\partial w}{\partial y} & \frac{\partial w}{\partial z} \end{pmatrix} \begin{pmatrix} x - x_0 \\ y - y_0 \\ z - z_0 \end{pmatrix}. \end{aligned} \tag{1}$$

Here we know that $\begin{pmatrix} u(x_0, y_0, z_0) \\ v(x_0, y_0, z_0) \\ w(x_0, y_0, z_0) \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$, and:

$$\mathbf{J} = \begin{pmatrix} \frac{\partial u}{\partial x} & \frac{\partial u}{\partial y} & \frac{\partial u}{\partial z} \\ \frac{\partial v}{\partial x} & \frac{\partial v}{\partial y} & \frac{\partial v}{\partial z} \\ \frac{\partial w}{\partial x} & \frac{\partial w}{\partial y} & \frac{\partial w}{\partial z} \end{pmatrix} \mathbf{x} = \begin{pmatrix} x - x_0 \\ y - y_0 \\ z - z_0 \end{pmatrix}. \tag{2}$$

Thus we get the linearized form of the magnetic vector field:

$$\frac{d\mathbf{x}}{dt} = \mathbf{J}\mathbf{x}. \tag{3}$$

Here to the first order approximation, a critical point can be classified according to the eigenvalues of the Jacobian matrix $\mathbf{J}$ of the vector $\mathbf{v}$ with respect to the critical point $\mathbf{x}$:

$$\left. \frac{\partial(u, v, w)}{\partial(x, y, z)} \right|_{x_0, y_0, z_0} = \left. \begin{pmatrix} \frac{\partial u}{\partial x} & \frac{\partial u}{\partial y} & \frac{\partial u}{\partial z} \\ \frac{\partial v}{\partial x} & \frac{\partial v}{\partial y} & \frac{\partial v}{\partial z} \\ \frac{\partial w}{\partial x} & \frac{\partial w}{\partial y} & \frac{\partial w}{\partial z} \end{pmatrix} \right|_{x_0, y_0, z_0}. \tag{4}$$

Figure 1 shows how the eigenvalues classify a critical point as an attracting node, a repelling node, an attracting focus, a repelling focus, and a saddle. This can be understood by observing that a positive or negative real part of an eigenvalue indicates an attracting or repelling nature, respectively. The imaginary part denoted circulation about the critical points. Among these points, the saddle points are distinct in that there are the characteristic magnetic Σ surfaces that are indicated in the figures and are spanned by the two eigenvectors that have the same sign of the real part of the eigenvalues. The outgoing or incoming characteristic two lines span the Σ surfaces.

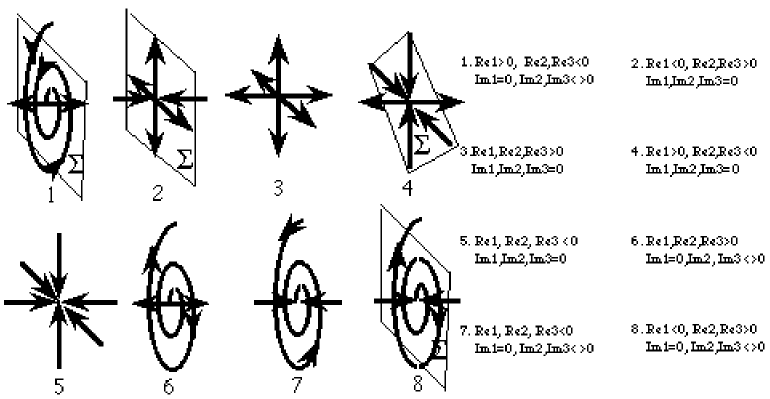

Figure 1: Classification of critical points.

In our magnetic vector field unlike the electric vector field we have the solenoidal condition $\nabla \cdot \mathbf{B} = 0$. Thus the three eigenvalues of the critical point $\lambda_1, \lambda_2, \lambda_3$ have the condition $\lambda_1 + \lambda_2 + \lambda_3 = 0$. Now we know that all the eigenvalues cannot be all positive or negative. We can only have the saddle points assuming that all the critical points are hyperbolic in the region we are visualizing. This observation gives us the unique global skeleton topology of the magnetic vector field.

3 Visualizing magnetic vector field topology in the earth magnetotail

The primary purpose of our visualization work is to develop a technique that leads to the global topological study of three-dimensional magnetic field reconnection. In this purpose, we studied the topology of the TRISTAN particle simulation [1] with the south interplanetary magnetic field (IMF). Although the TRISTAN code is the particle simulation and somehow noisy in the magnetic vector field data, we still be able to see the global magnetic vector field topology including the earth dipole magnetic field. In the following, we show the important magnetic field topology focusing on the two points 3 and 4 in Figs. 2 to 5. All the critical points found in visualizing the magnetotail and the earth magnetic field region are hyperbolic except the dipole. Thus it possible for us to determine the topology of the magnetic vector field in visualizing the region including the earth magnetic dipole that is indicated as the square box in the left side of the Figs. 2 to 5.

In the figures, the magnetic vector field curves are all characteristic ones that are starting from the critical points in the eigenvector directions. All surfaces shown in the figures are Σ surfaces, and are also called the separation surfaces that are spanned by the two eigenvectors that have the same sign of the real part of eigenvalues. The critical points 3 and 4 have the characteristic connections and connect to the sun and the earth, respectively, via the characteristic magnetic vector field curves.

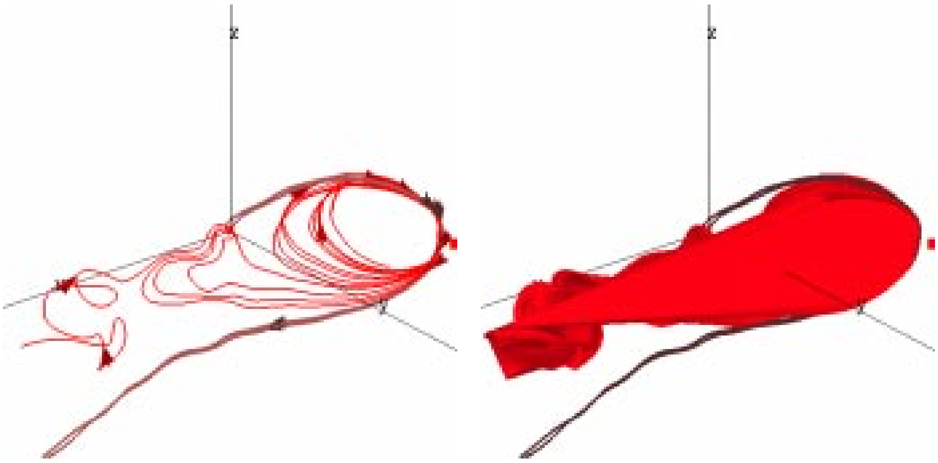

Figure 2: Topology of critical point 3.

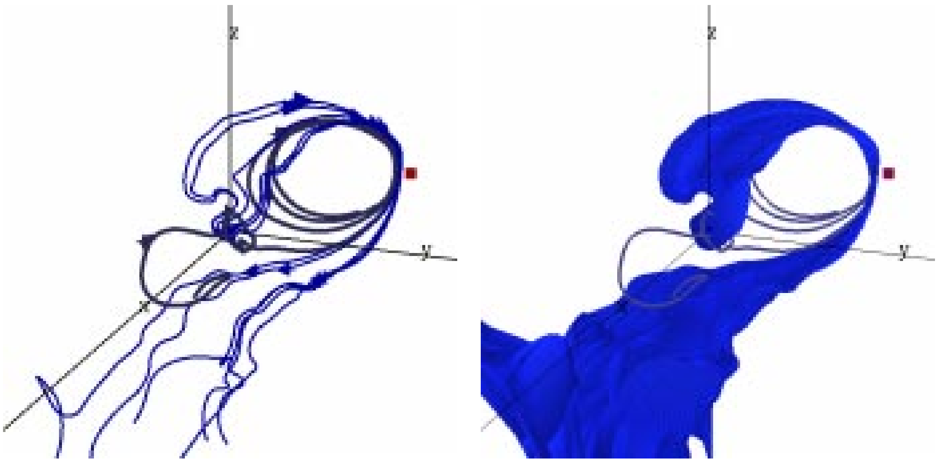

Figure 3: Topology of critical point 4.

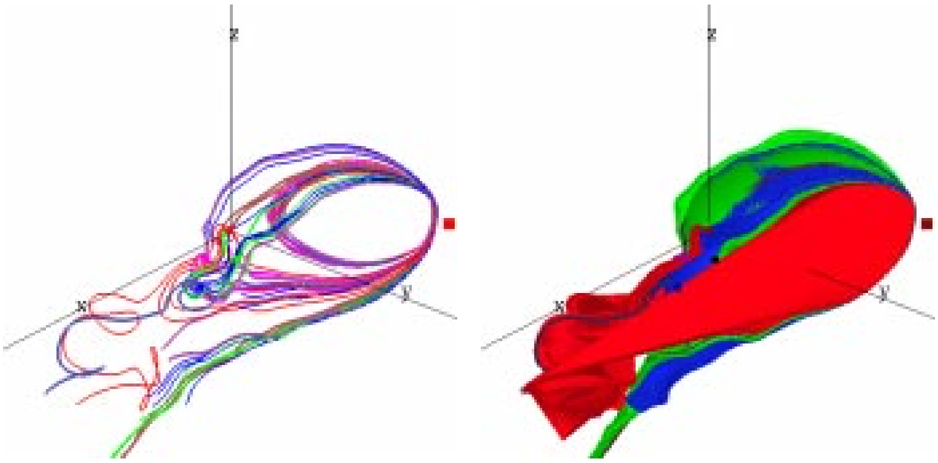

Figure 4: Whole topology in the magnetotail region including the earth magnetic dipole.

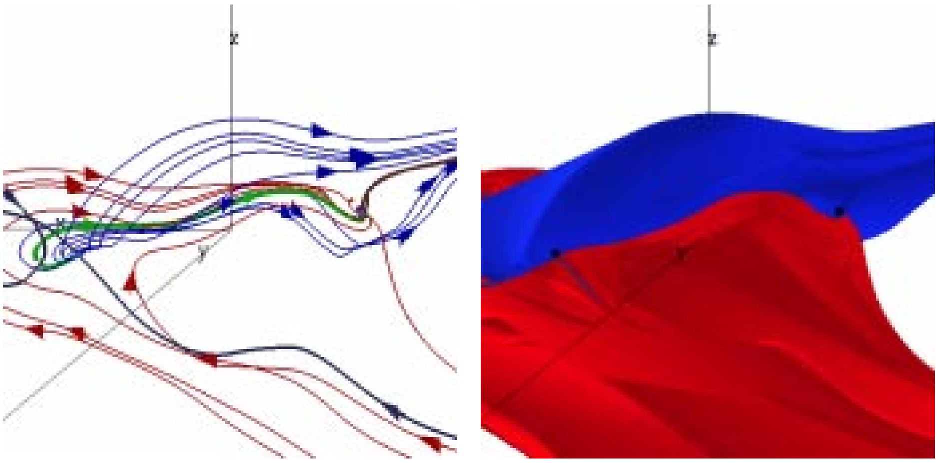

Figure 5: Complex three-dimensional X–point in the magnetotail and the earth magnetic field, which is the connection among critical points 3 and 4, and the earth dipole field.

References

[1] O. Buneman et al., Solar wind-magnetosphere interactions as simulated by a 3–D EM particle code, IEEE Trans. Plasma Sci., pp. 810–816, 20, 1992.

[2] V.I.Arnold, Ordinary differential equation, Springer,1981.